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# Compactness of Soft Rough Topological Space

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***Authors' contributions***

*This work was carried out in collaboration between both authors. Author LF designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. Authors LF and HF managed the analyses of the study. Author HF managed the literature searches. Both authors read and approved the final manuscript.*

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## Abstract

In this paper, the compactness of soft topological space is discussed in the soft rough formal context $\mathcal{T} = (G, M, R, F)$. The countable (finite) cover is defined over the rough soft formal context. Based on them, the soft compact topological space, compact subset, relative soft topology are defined, and compact properties of soft rough topological space is discussed over the rough soft formal context. The sufficient and necessary condition is given to check whether a given soft rough topological space is a compact.

*Keywords: Soft rough formal context; soft rough topological space; soft open(closed) set; countable(finite) cover; soft compactness.*

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# 1 Introduction

Topology [1, 2] can be formally defined as "the study of qualitative properties of certain objects (called topological spaces) that are invariant under a certain kind of transformation (called a continuous map). Topological spaces show up naturally in almost every branch of mathematics. This has made topology one of the great unifying ideas of mathematics.

Problems in many fields involve data that contain uncertainties. The uncertainty of data modeling the problems in engineering, physics, computer sciences, economics, social sciences, medical science and many other diverse fields. Uncertainties may be dealt with using a wide range of existing theories such as fuzzy set theory, theory of probability etc. Specially, formal concept context [3, 4], rough set [5, 6], soft set [7, 8] are extensively applied in the uncertainty reasoning. More and more researchers study these uncertainties(see[9-17]).

Some authors studied the topology with soft set and rough set, such as in [18], Shabir and Naz launched the study of soft topological spaces which was defined over an initial universe with a fixed set of parameters, and gave the concepts including soft open (closed) set, soft interior points and so on, they defined and discussed the soft $T_i$-space. In [19], authors continued investigating the properties of soft topological spaces. In [20], authors discussed the relationship among soft sets, soft rough sets and topologies.

In [21] , author defined the rough soft formal context, and discussed the rough properties of rough formal context in soft set. In [22], author defined some topological operators in the rough soft formal context, defined the soft rough topological space, and some topological properties were discussed over the soft rough topological space.

In this paper, we discuss the topological compactness of the soft rough formal context . The rest of this paper is organized as following. In section 2, we review some basic concepts and properties of rough concept formal soft sets, and soft rough topology , soft rough topological space over the soft rough formal context $\mathcal{T} = (G, M, R, F)$. In section 3, we study the compactness of soft rough topological space, and discuss compact properties of soft rough topological space over over the soft rough formal context $\mathcal{T} = (G, M, R, F)$. Conclusions are given in section 4.

# 2 Basic Knowledge

**Definition 2.1**[2] Let $\mathcal{A}$ be a collection of sets, $B$ be a set, if $B \subseteq \bigcup_{A\in\mathcal{A}} A$, then $\mathcal{A}$ is a **cover** of $B$; if $\mathcal{A}$ is a countable class or a finite class , then $\mathcal{A}$ is a **countable cover** or a **finite cover** of $B$; if $\mathcal{A}_1 \subseteq \mathcal{A}$, and $\mathcal{A}_1$ is a cover of $B$, then $\mathcal{A}_1$ is a **subcover** of $\mathcal{A}$ for $B$.

**Definition 2.2** [7] Let $U$ be an initial universe set and $E$ be a set of parameters. Let $\mathcal{P}(U)$ denotes the power set of $U$ and $A \subset E$. Then a pair $(F, A)$ is called a **soft set** over $U$, where $F : A \to \mathcal{P}(U)$ is a mapping.

That is, the soft set is a parameterized family of subsets of the set $U$. Every set $F(e), \forall e \in E$, from this family may be considered as the set of $e$**-elements** of the soft set $(F, E)$, or considered as the set of $e$**-approximate** elements of the soft set. According to this manner, we can view a soft set $(F, E)$ as consisting of collection of approximations: $(F, E) = \{F(e) \mid e \in E\} = \{(F(e), e) \mid e \in M\}$.

**Definition 2.3** [22] Let $(G, M, R)$ is a rough formal context, $G$ is objects set, is also called the universe, $M$ is attributes set. A pair $(F, B)$ is a soft set over $G$, where $B \subseteq M$, and $F : B \to \mathcal{P}(G)$ is a set-value mapping over $G$, furthermore, the lower and upper rough approximations of pair $(F, B)$ are denoted by $\underline{R}(F, B) = (\underline{F}, B), \overline{R}(F, B) = (\overline{F}, B)$, which are soft sets over $G$ with the set-valued mappings given by $\underline{F}(x) = \underline{B}(F(x))$ and $\overline{F}(x) = \overline{B}(F(x))$, where $x \in B$. The operators $\underline{R}, \overline{R}$ are called the **lower and upper rough approximation operators** on soft set $(F, B)$.

If $\overline{R} = \underline{R}$, we say that the soft set $(F, B)$ is **definable**, otherwise, $(F, B)$ is **rough**.

we call such quadruple tuple $(G, M, R, F)$ as **soft rough formal context**, and, such soft set $(F, B)$ on the soft rough formal context $(G, M, R, F)$ which is called **soft rough formal set**.

Obviously, $\forall x \in B \subseteq M, F(x) \subseteq G$ is a parameterized family of subsets of $G$, and $F(x)$ is the set of $x$- approximate elements in $(G, M, R, F)$.

**Definition 2.4** [22] Let $(G, M, R, F)$ be a soft rough formal context with the objects set $G$, and attributes set $M$. $B_1, B_2 \subseteq M$, $(F, B_1)$ and $(F_1, B_2)$ are two soft sets over $G$ on the soft rough formal context $(G, M, R, F)$. $F_1 : B \to \mathcal{P}(G)$ is a set-value mapping over the soft rough formal context $(G, M, R, F)$.

(i) If $B_1 \subseteq B_2$ , and $\forall x \in B_1 \subseteq B_2$, having $F(x) \subseteq F_1(x)$, then the soft sets $(F, B_1)$ is a **soft subset** of the soft set $(F_1, B_2)$, denoted as $(F, B_1)\widetilde{\subset}(F_1, B_2)$.

(ii) Two soft sets $(F, B_1)$ and $(F_1, B_2)$ on the rough soft formal context $(G, M, R, F)$ are said **soft equal** , if $(F, B_1)\widetilde{\subset}(F_1, B_2)$, and $(F_1, B_2)\widetilde{\subset}(F, B_1)$. We simply denote by $(F, B_1) = (F_1, B_2)$.

(iii) The **relative complement** of $(F, B)$ is denoted by $(F, B)^c$ and is defined by $(F, B)^c = (F^c, B)$, where $F^c : B \to \mathcal{P}(G)$, and $F^c(x) = G - F(x), \forall x \in B$.
Clearly, $((F, B)^c)^c = (F, B)$.

(iv) $(F, B)$ is said to be a **relative null soft rough formal set** denoted by $\mathcal{N}$, if $\forall x \in B, F(x) = \emptyset$; if $B = M$, then is called **absolute null soft rough formal set**, denoted as $\widetilde{\emptyset}$.

(v) $(F, B)$ is said to be a **relative whole soft rough formal set** denoted by $\widetilde{G}$, if $\forall x \in B, F(x) = G$.

**Definition 2.5**[22] Let $(G, M, R, F)$ be the soft rough formal context, $(F_1, B_1)$ and $(F_2, B_2)$ are two rough soft formal sets over $(G, M, R, F)$, in which $F_1, F_2 : B \to \mathcal{P}(G)$ are two set-value mappings.

(i) The **union** of $(F_1, B_1)$ and $(F_2, B_2)$ is the rough soft formal set $(H, C)$, where $C = B_1 \cup B_2$, and $\forall e \in C$, denoted as $(F_1, B_1)\widetilde{\cup}(F_2, B_2) = (H, C) = (H, B_1 \cup B_2)$, where

$$H(e) = \begin{cases} F_1(e), & if \quad e \in B_1 - B_2 \\ F_2(e), & if \quad e \in B_2 - B_1 \\ F_1(e) \cup F_2(e), & if \quad e \in B_1 \cap B_2 \end{cases}$$

(ii) The **intersection** of $(F_1, B_1)$ and $(F_2, B_2)$ is the soft rough formal set $(H, C)$ is denoted as $(F_1, B_1) \cap (F_2, B_2)$ and is defined as $(F_1, B_1) \sqcap (F_2, B_2) = (H, C)$, where $C = B_1 \cap B_2$, and $\forall e \in C, H(e) = F_1(e) \cap F_2(e)$.

**Definition 2.6**[23] Let $\mathcal{T} = (G, M, R, F)$ be a soft rough formal context over the object set $G$ and attributes set $M$, $B_i \subseteq M$, $\tau = \{(F_i, B_i) \mid (F_i, B_i)$ is a soft set over $G\}$ which is the collection of soft sets on the soft rough formal context $(G, M, R, F)$, if

(1) $\widetilde{\emptyset}, \widetilde{G}$ belong to $\tau$.
(2) The union of any number of soft sets in $\tau$ belongs to $\tau$, that is, $\tau$ is closed for the any union of soft sets over $\mathcal{T}$.
(3) The intersection of any two soft sets in $\tau$ belongs to $\tau$, that is, $\tau$ is closed for the finite intersection of soft sets over $\mathcal{T}$.

Then the collection $\tau$ is calleda **soft topology** over the rough soft formal context $\mathcal{T}$(simply called **soft rough topology**). The triplet $(G, \tau, M)$ is called **a soft topological space** over the soft rough formal context $\mathcal{T}$ (simply called **soft rough topological space**).

The members of $\tau$ are **soft open** sets in $\mathcal{T}$, the relative complement $(F, B)^c = (F^c, B)$ is said to be a **soft closed** set in $\mathcal{T}$ if $(F, B)^c \in \tau$.

# 3 Compact Topological Space Over the Soft Rough Formal Context

As the following goes, our study is based on the soft rough formal context $\mathcal{T} = (G, M, R, F)$ in which $G$ is an initial universe set, $M$ is a set of attributes, $R \subseteq G \times M$. Firstly, we classify data using given relation $R$, getting upper(lower) approximation of $G$, that is , considering its rough properties; secondly, we consider soft sets which are generated by upper(lower) approximation, finally, we achieve the collection of soft sets.

Suppose that we have finished the above working, and $(G, \tau, M)$ is a soft rough topological space over $\mathcal{T}$, $(F, B)$ is a soft set in $\tau$. Then, we simply say $(G, \tau, M)$ be a soft rough topological space over $\mathcal{T}$.

**Note** In reference[18-20], authors all discussed soft topological space, in particular, authors defined the soft compactness in [20], in this paper, we also study the soft compactness of soft topology in another way and in a different context, that is, we base on the soft rough formal context, and we discuss the soft compactness of soft rough topological space.

**Definition 3.1** Let $(G, \tau, M)$ be a soft rough topological space over $\mathcal{T}$, $(F, B)$ be a soft set over $G$, and $x \in G$. we say $x \in (F, B)$ and read as $x$ **belongs to** the soft set $(F, B)$, if $x \in F(e)$ for all $e \in B$; and if there is some $e \in B$, such that $x \notin F(e)$, then $x \notin (F, B)$, read as $x$ does not belong to the soft set $(F, B)$.

**Definition 3.2** Let $(G, \tau, M)$ be a soft rough topological space over $\mathcal{T}$ $(F, B)$ be a soft set over $G$, and the collection of soft sets $\mathcal{A} = \{(F, B) \mid (F, B)$ is soft open (closed), $(F, B) \in \tau\}$ is a cover of $G$, then $\mathcal{A}$ is a soft **open(closed) cover** of $G$.

**Definition 3.3** Let $(G, \tau, M)$ be a soft rough topological space over $\mathcal{T}$, $(F, B)$ be a soft set over $\mathcal{T}$, and the collection of soft sets $\mathcal{A} = \{(F, B) \mid (F, B)$ is an soft open set in $\tau\}$ be any soft open cover of $G$, if $\mathcal{A}$ has a soft countable subcover of $G$, then $(G, \tau, M)$ is a **soft rough Lindelöff topological space** over $\mathcal{T}$.

Obviously, let $(G, \tau, M)$ be a discrete soft rough topological space over $\mathcal{T}$, and $(F, B)$ be a soft set over $G$, suppose that the soft rough topology $\tau = \{(F_i, M) \mid F_i : M \to \wp(G)\}$ is a discrete soft rough topology over $\mathcal{T}$. Then $(G, \tau, M)$ is not a soft rough Lindelöff topological space over $\mathcal{T}$.

**Definition 3.4** Let $(G, \tau, M)$ be a soft rough topological space over $\mathcal{T}$ $(F, B)$ is a soft set over $\mathcal{T}$, and the collection of soft sets $\mathcal{A} = \{(F, B) \mid (F, B)$ is soft open set in $\tau\}$ be any soft open cover of $G$, if $\mathcal{A}$ has a soft finite sub-collection which covers $G$, then $(G, \tau, M)$ is a **soft rough compact topological space** over $\mathcal{T}$, simply say, $(G, \tau, M)$ is a **soft rough compact**.

Clearly, if $(G, \tau, M)$ is a soft rough compact topological space over $\mathcal{T}$, then it must be a softrough Lindelöff topological space over $\mathcal{T}$, however, the converse does not hold.

For example, let $(G, \tau, M)$ be a discrete soft rough topological space over $\mathcal{T}$, and $G$ be an initial universe set which includes countable infinite points, $M$ be a set of attributes, and $(F, B)$ is a soft set over $G$, then collection of soft sets $\tau = \{(F_i, M) \mid F_i : M \to \wp(G)\}$ forms a discrete soft rough topology over $\mathcal{T}$, and $(G, \tau, M)$ is a soft rough Lindelöff topological space over $\mathcal{T}$, but it is not the soft rough compact topological space over $\mathcal{T}$.

**Example 1** Let $(G, \tau, M)$ be a soft rough topological space over $\mathcal{T}$, where $G$ is real number set $\mathbb{R}, M$ is a set of attributes, $B \subseteq M$. $F : B \to \mathcal{P}(G)$ is the set-value mapping over $G$, such that $\forall e \in M, F(e) = (-n, n) \subseteq \mathbb{R}$, and the pair $(F, B) = (e, \{(-n, n) \mid n \in \mathbb{Z}^+\})$ is soft open set over $G$, then the collection of soft sets $\tau = \{(F, B) \mid F : B \to \mathcal{P}(G), \forall e \in B, F(e) \subseteq \mathbb{R}\}$ is a soft rough topology over $\mathcal{T}$, and $\tau$ is a soft cover of $G$.

Taking $\tau_1 = \{(F_i, B_i) \mid (F_i, B_i)$ is a soft set over $G\}$, in which $F_i : B_i \to \mathcal{P}(G)$ is the set-value mapping over $G$, such that $F_i(e) = (-n_i, n_i) \subseteq \mathbb{R}, B_i \subseteq B, i \in \mathbb{Z}^+$, define the union of $(F_1, B_1)$ and $(F_2, B_2)$ as $(F_1, B_1)\tilde{\cup}(F_2, B_2) = (H, C) = (H, B_1 \cup B_2)$, such that $\forall e \in C = B_1 \cup B_2$, having

$$H(e) = \begin{cases} F_1(e), & if \;\; e \in B_1 - B_2 \\ F_2(e), & if \;\; e \in B_2 - B_1 \\ F_1(e) \cup F_2(e) & if \;\; e \in B_1 \cap B_2 \end{cases}$$

where $F_1(e) \cup F_2(e) = \{(-max\{n_1, n_2\}, max\{n_1, n_2\}) \mid n_1, n_2 \in \mathbb{Z}^+\}$, then $\tau_1 \subseteq \tau$ is a soft subset of $G$.

Clearly, $\tau_1$ is not the soft subcover of $G = \mathbb{R}$, that is, $G = \mathbb{R}$ does not have any finite subcover. Hence, $(G, \tau, M)$ is not soft compact topological space over $\mathcal{T}$.

However, we can prove that the soft rough topological space with the object set $[0, 1]$, in which the soft set $(F, B), F : B \to \mathcal{P}([0, 1])$, such that, $\forall e \in B \subseteq M, F(e) \subset [0, 1]$, if $([0, 1], \tau, M)$ is a soft rough topological space, then $([0, 1], \tau, M)$ is a soft compact.

**Proof** Suppose that the collection $\mathcal{A} = \{(F, B)|(F, B) \in \tau)\}$ is an soft open cover of $[0, 1]$, $\mathcal{P} = \{A_1, A_2, \cdots, A_n\}$ is a sub-collection of $\mathcal{A}$ which is covers $[0, x], \forall x \in [0, 1]$, obviously, $\mathcal{P} \neq \emptyset$, we only need to prove $\mathcal{P}$ also is a soft open cover of $[0, 1]$.

Without loss generality, taking $x \in [0, 1)$, then $\exists 1 \leq i_0 \leq n$, such that $x \in A_{i_0} = (F_{i_0}, B) \in \tau$, in which $\forall e \in B, F_{i_0}(e) \subseteq \mathcal{P}([0, 1])$, that is , $x \in F_{i_0}(e) \subseteq \mathcal{P}([0, 1])$, then $\forall \varepsilon > 0, [x, x + \varepsilon) \subset A_{i_0}$, and $[0, x + \varepsilon) \subset \bigcup\limits_{i=1}^{n} A_i$.

Hence, $\mathcal{P}$ is a soft open cover of $[0, 1]$.

**Definition 3.5** Let $(G, \tau, M)$ be a soft rough topological space over $\mathcal{T}$ $(F, B)$ is a soft set over $\mathcal{T}$, and the collection of soft sets $\mathcal{A} = \{(F, B) \mid (F, B)$ is soft open set in $\tau\}$ be any soft countable open cover of $G$, if $\mathcal{A}$ has a soft finite sub-collection which covers $G$, then $(G, \tau, M)$ is a **soft rough countable compact topological space** over $\mathcal{T}$, simply say, $(G, \tau, M)$ is a **soft rough countable compact**.

Clearly, by this definition, we have:

**Proposition 3.1** Let $(G, \tau, M)$ be any soft rough compact topological space over $\mathcal{T}$, then it is also soft rough countable compact.

**Proposition 3.2** Let $(G, \tau, M)$ be any soft rough Lindelöff countable compact topological space over $\mathcal{T}$, then it is also soft rough compact.

**Definition 3.6** Let the triplet $(G, \tau, M)$ be a soft rough topological space over $G$, the collection soft open sets $\tau$ is a soft rough topology over $G$, and $A \subseteq G$, define $(F, B) \cap A = (e, F(e)) \cap A = (e, F(e) \cap A)$, where $(F, B) \in \tau, e \in B$, denote as $(F, B)|_A = (F|_A, B)$, and $F|_A : B \to \mathcal{P}(G) \cap A, \forall e \in B, F|_A(e) = F(e) \cap A \subseteq A$ which is called **refinement** of $(F, B)$ on $A$.

Correspondingly, denote the soft rough topology decided by the soft set $(F|_A, B)$ as $\tau \mid_A = \{(F|_A, B) \mid (F, B) \in \tau\}$.

**Lemma 3.3** Let $(G, \tau, M)$ be a soft rough topological space over $G$, the collection soft open sets $\tau = \{(F, B) \mid F : B \to \mathcal{P}(G), \forall e \in B, F(e) \subseteq G\}$ be a soft topology over $G$, and $A \subseteq G$, $\tau \mid_A = \{(F|_A, B) \mid F|_A : B \to \mathcal{P}(G) \cap A, \forall e \in B, F|_A(e) = F(e) \cap A \subseteq A\}$, then $(G, \tau \mid_A, M)$ is also a soft rough topology of $A$.

**Proof** (i) By $\tau$ being a soft topology over $G$, so $\widetilde{G} = \{(F, B) \mid \forall e \in B, F(e) = G\} \in \tau$, and $A \subseteq G$, then $A = G \cap A$, having $\widetilde{A} = \{(F \mid_A, B) \mid \forall e \in B, F|_A(e) = A, (F, B) \in \tau\} \in \tau|_A$;
Similarly, $\widetilde{\emptyset} = \{(F, B) \mid \forall e \in B, F(e) = \emptyset \subseteq G\} \in \tau$, by $\emptyset = G \cap \emptyset$, then $\widetilde{\emptyset} = \{(F \mid_A, B) \mid (F, B) \in \tau, \forall e \in B, F|_A(e) = \emptyset \cap A = \emptyset\} \in \tau|_A$;

(ii) Taking $(F_1|_A, B_1), (F_2|_A, B_2) \in \tau|_A$, then there exist soft sets $(F_1, B_1), (F_2, B_2) \in \tau$, such that $\forall e \in B_1$, or $\forall e \in B_2, F_1|_A(e) = F_1(e) \cap A \subseteq A, (F_2|_A(e) = F_2(e) \cap A \subseteq A$, by $\tau$ is a soft topology, then $(e, F_1(e) \cap F_2(e)) \in \tau$, so $(F_1|_A, B_1) \sqcap (F_2|_A, B_2) = (H|_A, B_1 \cap B_2)$, $\forall e \in B_1 \cap B_2, H|_A(e) = F_1|_A(e) \cap F_2|_A(e) = (F_1(e) \cap F_2(e)) \cap A$, that is, $(F_1|_A, B_1) \sqcap (F_2|_A, B_2) \in \tau|_A$;

(iii) Let $\tau_1 \subseteq \tau|_A$, then $\forall (F|_A, B) \in \tau_1 \subseteq \tau|_A, \exists (F, B) \in \tau$, such that $\forall e \in B, F|_A(e) = F(e) \cap A$, by the definition of soft union, $\widetilde{\bigcup}_{(F|_A, B) \in \tau_1} (F|_A, B) = \widetilde{\bigcup}_{(F|_A, B) \in \tau_1} (e, F(e) \cap A) = (e, \bigcup_{(F|_A, B) \in \tau_1} (F(e) \cap A))$, by $\tau$ is a soft rough topology, then $(e, \bigcup_{(F,B) \in \tau} F(e)) \in \tau$. so, $\widetilde{\bigcup}_{(F|_A, B) \in \tau_1} (F|_A, B) \in \tau_1$.
Hence, $(G, \tau \mid_A, M)$ is also a soft rough topology of $A$.

**Definition 3.8** Let the triplet $(G, \tau, M)$ be a soft rough topological space over $\mathcal{T}$, the collection soft open sets $\tau$ is soft topology over $G$, and $A \subseteq G$, the soft topology of $A$(denoted $\tau \mid_A$) is a **relative soft rough topology** ( relative to $\tau$ of $G$), and soft rough topological space $(G, \tau \mid_A, M)$ is a soft rough topological **subspace** of $(G, \tau, M)$ over $G$.

**Definition 3.9** Let the triplet $(G, \tau, M)$ be a soft rough topological space over $\mathcal{T}$, the collection soft open sets $\tau$ be a soft topology over $G$, and $A \subseteq G$, soft rough topological space $(G, \tau \mid_A, M)$ be a soft rough topological subspace of $(G, \tau, M)$, and if $(G, \tau \mid_A, M)$ is compact, then $A$ is a **compact subset** of $G$.

**Theorem 3.4** Let the triplet $(G, \tau, M)$ be a soft rough topological space over $\mathcal{T}$, the collection soft open sets $\tau$ is soft topology over $G$, and $A \subseteq G, (G, \tau \mid_A, M)$ is a soft topological subspace of $(G, \tau, M), \mathcal{A} = \{(F, B) | (F, B)$ is soft open set over $(G, \tau, M)\}$, and $\mathcal{A}$ is a soft cover of $A$, then $A$ is a compact subset of $G$ if and only if $\mathcal{A}$ has a finite soft subcover.

**Proof** Let $A \subseteq G$, the collection of soft sets $\mathcal{A} = \{(F, B) | (F, B)$ be soft open set over $(G, \tau, M)\}$, and $A$ be a compact subset of $G$.

Define the collection $\widetilde{\mathcal{A}} = \{(F|_A, B) | F|_A : B \to \mathcal{P}(G) \cap A, (F, B)$ be soft open set over $(G, \tau, M)\}$, then $\widetilde{\mathcal{A}}$ is also a soft cover of $A$, and the member of $\widetilde{\mathcal{A}}$ is a soft open set of $(G, \tau \mid_A, M)$, the collection $\{(e_1, F(e_1) \cap A), (e_2, F(e_2) \cap A), \cdots, (e_n, F(e_n) \cap A)\}$ is a soft finite subcover of $\widetilde{\mathcal{A}}$, that is , $\widetilde{\mathcal{A}}$ has a finite subcover.

On the other hand, suppose that $\mathcal{A}$ has a finite subcover $\widetilde{\mathcal{A}}$. For any soft set $(F, B) \in \mathcal{A}$, there exists an open set $U_A = (F_A, B) \in \widetilde{\mathcal{A}}$, such that $\forall e \in B, F(e) = F_A(e) \cap A$, then $\widetilde{\mathcal{A}} = \{(F|_A, B) | (F, B) \in \tau, \forall e \in B, F(e) = F_A(e) \cap A\}$ is a soft subcover of $A$, and $\{(F_1, B), (F_2, B), \cdots, (F_n, B)\}$ is a

sub-collection of $\mathcal{A}$, so $\widetilde{\mathcal{A}}$ has a finite soft subcover $\{(F|_{A_1}, B), (F|_{A_2}, B), \cdots, (F|_{A_n}, B)\}$, in which $\forall e \in B, F|_{A_i}(e) = F_i(e) \cap A, i = 1, 2, \cdots n$.
Hence $A$ is a compact subset of $G$.

**Definition 3.10** Let $A \subseteq G$ be a soft rough topological space over $\mathcal{T}$, the collection of soft sets $\mathcal{A} = \{(F, B)|(F, B)$ be a soft set in $(G, \tau, M)\}$ , if every finite sub-collection of $\mathcal{A}$ has a non-empty intersection (that is, if $\mathcal{A}_1 \subseteq \mathcal{A}$, then $\sqcap_{(F,B)\in\mathcal{A}_1}(F, B) \neq \widetilde{\emptyset}$), then $\mathcal{A}$ is the collection **having finite intersection** in $(G, \tau, M)$.

**Proposition 3.5** Let $(G, \tau, M)$ be a soft rough topological space over $\mathcal{T}$, $(G, \tau, M)$ is a soft rough compact space if and only if every finite sub-collection of soft closed sets $\mathcal{A} = \{(F, B)|(F, B)$ is a soft closed set in $(G, \tau, M)\}$ has non-empty intersection.

**Proof** Let $(G, \tau, M)$ be a soft rough compact topological space over $\mathcal{T}$. Suppose that the collection of soft closed sets $\mathcal{F} = \{(F, B)|(F, B)$ is a closed soft set in $\tau\}$ has non-empty intersection. We should prove $\sqcap_{(F,B)\in\mathcal{F}}(F, B) \neq \emptyset$.

(i) If $\mathcal{F} = \emptyset$, then $\sqcap_{(F,B)\in\mathcal{F}}(F, B)$ makes no sense.

(ii) If $\mathcal{F} \neq \emptyset$, and $\sqcap_{(F,B)\in\mathcal{F}}(F, B) = \widetilde{\emptyset}$, that is, $(e, \cap_{e\in B}F(e)) = (e, \emptyset)$, i.e. $\cap_{e\in B}F(e) = \emptyset$.

Let $\mathcal{A} = \{(F, B)^c|(F, B) \in \mathcal{F}\}$, by $\widetilde{\cup}_{(F,B)\in\mathcal{F}}(F, B)^c = (\sqcap_{(F,B)\in\mathcal{F}}(F, B))^c = ((e, \cap_{i\in B}F(e))^c = (e, G - \cap_{e\in B}F(e)) = (e, G) = \widetilde{G}$, then $\mathcal{A}$ is a soft open cover of $G$, and $\mathcal{A}$ has a finite soft subcover, denote as $\{(F_1, B_1)^c, (F_2, B_2)^c, \cdots, (F_n, B_n)^c\}$, where $(F_i, B_i) \in \tau$,
$B_i \subseteq M, i = 1, 2, \cdots n$, without loss generality, we take the same $B$, then $(F_1, B) \sqcap (F_2, B) \sqcap \cdots \sqcap (F_n, B) = (e, \bigcap\limits_{i=1}^{n} F_i(e)) = (e, \bigcup\limits_{i=1}^{n} F_i^c(e))^c = ((F_1, B)^c\widetilde{\cup}(F_2, B)^c\widetilde{\cup}\cdots\widetilde{\cup}(F_n, B)^c)^c = \widetilde{G}^c = \widetilde{\emptyset}$.

Conversely, if every finite sub-collection of soft closed sets $\mathcal{A} = \{(F, M)|(F, M)$ is a soft closed set in $(G, \tau, M)\}$ has non-empty intersection, let $\mathcal{A} = \{(F, B)|(F, B) \in \tau\}$ be a collection of soft open sets, we need to show $\mathcal{A} = \{(F, M)|(F, M)$ is a soft closed set in $(G, \tau, M)\}$ has a finite subcover.

(i) If $\mathcal{A} = \emptyset$, then $\widetilde{\cup}_{(F,B)\in\mathcal{A}}(F, B) = (e, \cup_{e\in B}F(e)) = (e, \emptyset) = \widetilde{\emptyset}$, it implies $G = \emptyset$, and any sub-collection of $\mathcal{A}$ is a cover of $G$.

(ii) If $\mathcal{A} \neq \emptyset$, let $\mathcal{F} = \{(F, B)^c|(F, B) \in \mathcal{A}\}$ be an non-empty soft closed sub-collection of $\widetilde{G}$, and $\sqcap_{(F,B)^c\in\mathcal{F}}(F, B)^c = (e, \bigcap\limits_{i=1}^{n} F_i^c(e)) = (e, \bigcup\limits_{i=1}^{n} F_i(e)) = \widetilde{\cup}_{(F,B)\in\mathcal{A}}(F, B) = \widetilde{\emptyset}$ which does not have non-empty intersection, that is, $\mathcal{F}$ has a finite sub-collection, but the intersection of the members of this finite sub-collection is an empty set.

Suppose that this finite sub-collection is $\{(F_1, B)^c, (F_2, B)^c, \cdots, (F_n, B)^c\}$, then $(F_1, B)^c\sqcap(F_2, B)^c\sqcap \cdots \sqcap (F_n, B)^c = (e, \bigcap\limits_{i=1}^{n} F_i^c(e)) = (e, \emptyset)$, so $(F_1, B)\widetilde{\cup}(F_2, B)\widetilde{\cup}\cdots\widetilde{\cup}(F_n, B) = (e, \bigcup\limits_{i=1}^{n} F_i(e)) = \widetilde{G}$, that is $\{(F_1, B), (F_2, B), \cdots, (F_n, B)\}$ is a finite subcover of $\mathcal{A}$.

**Proposition 3.6** If $(G, \tau, M)$ is a soft rough compact topological space, $(G_1, B)$ is a sot closed set over $(G, \tau, M)$, that is $G_1 : B \to \mathcal{P}(G), \forall e \in B, G_1(e) \subseteq G$, $G_1(e)$ is a closed subset of $G$, then $(G_1, B)$ is a soft compact subset of $\widetilde{G}$.

**Proof** Supposed that $(G_1, B)\widetilde{\subset}\widetilde{G}$, $(G_1, B)$ is closed. Let $\mathcal{A} = \{(F, B)|(F, B) \in \tau\}$ be a cover of $(G_1, B)$, then $\mathcal{B} = \mathcal{A} \cup \{(G_1, B)^c\}$ is an open cover of $\widetilde{G}$. Let $\mathcal{B}_1 \subseteq \mathcal{B}$ be a finite sub-collection of $\mathcal{B}$, and $\mathcal{B}_1$ be a cover of $\widetilde{G}$, then $\mathcal{B}_1 - \{(G_1, B)^c\} \subseteq \mathcal{A}$ is a finite sub-collection of $\mathcal{A}$ which is a cover of

$(G_1, B)$. Hence, $(G_1, B)$ is a soft compact subset of $\widetilde{G}$.

**Proposition 3.7** If $(G, \tau, M)$ is a soft rough topological space, then $(F_1, B), (F_2, B), \cdots, (F_n, B) \in (G, \tau, M)$, and $(F_1, B), (F_2, B), \cdots, (F_n, B)$ are soft compact subsets of $\widetilde{G}$, then $\widetilde{\bigcup}_{i=0}^{n}(F_i, B)$ is a soft compact subset of.

**Proof** Suppose that $(F_i, B), i = 1, 2 \cdots, n$ is soft compact subsets of $\widetilde{G}$, denote $(F, B) = \widetilde{\bigcup}_{i=0}^{n}(F_i, B) = (e, \bigcap\limits_{i=1}^{n} F_i(e))$, let $\mathcal{A} = \{(F, B)|(F, B) \in (G, \tau, M)\}$ be an open cover of $\widetilde{G}$, then $\mathcal{A}$ is an open cover of $(F_i, B), i = 1, 2 \cdots, n$, by $(F_i, B)$ is soft compact subsets of $\widetilde{G}$, so there is a finite subcover of $(F_i, B)$, denoted by $\mathcal{B}_i$, taking $\mathcal{A}_1 = \bigcup\limits_{i=1}^{n} \mathcal{B}_i$, then $\mathcal{A}_1$ is a finite subcover of $(F, B)$.

Hence, $\bigcup\limits_{i=0}^{n}(F_i, B)$ is a soft compact subset of $\widetilde{G}$.

Similarly, by the definition of soft rough topological space, the finite intersection of soft sets ia a soft set over $(G, \tau, M)$, hence, for soft closed sets , we have:

**Proposition 3.8** If $(G, \tau, M)$ is a soft rough topological space, then $\mathcal{F} = \{(F, B)|(F, B)$ is a soft compact closed set $\in (G, \tau, M)\}$, that is, soft compact closed set $(F, B)\widetilde{\subset}\widetilde{G}$, then $\sqcap_{(F,B)\in\mathcal{F}}(F, B) = (e, \bigcap\limits_{i=1}^{n} F_i(e))$ is a soft compact subset of $\widetilde{G}$.

**Theorem 3.9** If $(G, \tau, M)$ is a soft rough topological space, then $(G, \tau, M)$ is an open subspace of a soft rough compact topological space.

**Proof** Let $(G, \tau, M)$ be a soft rough topological space, $\infty \notin G$, $G^* = G \cup \{\infty\}, \tau^* = \tau \cup \tau_1 \cup \widetilde{X^*})$, where $\tau_1 = \{(F, B) \sqsubseteq \widetilde{X^*} | \widetilde{X^*} - (F, B)$ is a soft compact closed set in $(G, \tau, M)\}$.

**First**, show that $(G^*, \tau^*, M)$ is a soft rough topological space.

(i) By the definitions of $G^*, \tau^*$, having $\widetilde{G^*} \in \tau^*, \emptyset \in \tau \subseteq \tau^*$.

(ii) Let $(F_1, B)^*, (F_2, B)^* \in \tau^*$, if $(F_1, B)^*$ or $(F_2, B)^*$ is $\widetilde{G^*}$, clearly, $(F_1, B)^* \sqcap (F_2, B)^*$ is $(F_1, B)^*$ or $(F_2, B)^*$ and belongs to $\tau^*$.

Next, suppose that $(F_1, B)^*, (F_2, B)^* \neq \widetilde{G^*}$, then there three cases.

Case 1: If $(F_1, B)^*, (F_2, B)^* \in \tau$, then $(F_1, B)^* \sqcap (F_2, B)^* \in \tau \subseteq \tau^*$.

Case 2: If $(F_1, B)^*, (F_2, B)^* \in \tau_1$, then $\widetilde{G^*} - ((F_1, B)^* \sqcap (F_2, B)^*) = (\widetilde{G^*} - (F_1, B)^*)\widetilde{\cup}(\widetilde{G^*} - (F_2, B)^*) \in \tau \subseteq \tau^*$ is the union of soft compact subsets which is also a soft compact subset, so,$(F_1, B)^* \sqcap (F_2, B)^* \in \tau_1 \subset \tau$.

Case 3: If both case 1 and case 2 do not hold. Supposed that $(F_1, B)^* \in \tau_1, (F_2, B)^* \in \tau$, then $(F_1, B)^* \sqcap (F_2, B)^* \in \tau \subset \tau^*$

(iii) Let $\mathcal{A} \subseteq \tau^*$, and $\widetilde{\cup}_{(F,B)\in\mathcal{A}}(F, B) \neq \widetilde{\emptyset}$ or $\widetilde{G^*}$, then $\mathcal{A} \neq \widetilde{\emptyset}$ or $\widetilde{G^*} \notin \mathcal{A}$, and there are three cases.

Case 1 If $\mathcal{A} \subseteq \tau$, obviously, $\widetilde{\cup}_{(F,B)\in\mathcal{A}}(F, B) \in \tau \subset \tau_1$.

Case 2 If $\mathcal{A} \subseteq \tau_1$, then $\widetilde{G^*} - \widetilde{\cup}_{(F,B)\in\mathcal{A}}(F, B) = \sqcap(\widetilde{G^*} - (F, B))\widetilde{\subset}(\widetilde{G^*} - (F_0, B))$ is closed, where $\forall(F_0, B) \in \mathcal{A}$.

Case 3 If both case 1 and case 2 do not hold, then $\mathcal{A}_1 = \mathcal{A} \sqcap \tau \neq \widetilde{\emptyset}$, $\mathcal{A}_2 = \mathcal{A} \sqcap \tau_1 \neq \widetilde{\emptyset}$. Suppose $(F_1, B) = \widetilde{\cup}_{(F,B)\in\mathcal{A}_1}(F, B), (F_2, B) = \widetilde{\cup}_{(F,B)\in\mathcal{A}_2}(F, B)$, then $\widetilde{\cup}_{(F,B)\in\mathcal{A}}(F, B) = (F_1, B)\widetilde{\cup}(F_2, B)$, and $(F_1, B) \in \tau$,
$(F_2, B) \in \tau_1$, having $\widetilde{G^*} - ((F_1, B)\widetilde{\cup}(F_2, B)) = (\widetilde{G^*} - (F_1, B)) \sqcap (\widetilde{G^*} - (F_2, B))$ is closed, so, $\widetilde{\cup}_{(F,B)\in\mathcal{A}}(F, B) \in \tau \subset \tau^*$.

Hence,$(G^*, \tau^*, M)$ is a soft rough topological space.

**Second**, prove that $(G^*, \tau^*, M)$ is soft rough compact.

Suppose $\mathcal{C}$ is an soft open cover of $\widetilde{G^*}$, then $\exists(F, B) \in \mathcal{C}$, such that $\infty \in (F, B)$, and $(F, B) \in \tau_1$, so $\widetilde{G^*} - (F, B)$ is soft compact, and $\mathcal{C} - (F, B)$ is its soft open cover, then $\mathcal{C} - (F, B)$ has a finite sub-collection, denoted as $\widetilde{\mathcal{C}}$ which cover $\mathcal{G} - (F, B)$, and $\widetilde{\mathcal{C}}\widetilde{\cup}(F, B)$ which cover $\widetilde{G^*}$ is a finite sub-collection of $\mathcal{C}$.

**Finally**, by $\tau = \tau^*|_{G^*}, \widetilde{G}$ is a soft open set of $\widetilde{G^*}$, so, $(G, \tau, M)$ is an soft rough open subspace of $(G^*, \tau^*, M)$.

# 4 Conclusion

In this paper, we study the compactness of soft rough topological space, and discuss compact properties of soft rough topological space over the soft rough formal context $\mathcal{T} = (G, M, R, F)$. And we define the countable (finite) cover, we discuss the soft compact topological space, compact subset, relative soft topology are defined, and compact properties of soft rough topological space, over the rough soft formal context, and we give a sufficient and necessary condition to check whether a given soft rough topological space is a compact. That is, we investigate the compactness of soft rough topological space which offers a new method and tool in data analysis . We will discuss the relationship between compact soft topological space and topological separate axioms over the soft rough formal context $\mathcal{T} = (G, M, R, F)$ later.

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# Competing Interests

Authors have declared that no competing interests exist.

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