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Mathematical Model of Drinking Epidemic

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Authors' contributions

This work was carried out in collaboration between all authors. Authors IKA and MARENO assisted in the writing of the draft, numerical simulations and review of the final draft. Author CY developed the model equations and review the final draft. All authors read and approved the final manuscript.

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Abstract

A non-linear $SHTR$ mathematical model was used to study the dynamics of drinking epidemic. We discussed the existence and stability of the drinking-free and endemic equilibria. The drinking-free equilibrium was locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$. Global stability of drinking-free and endemic equilibria were also considered in the model, using Lassalle's invariance principle of Lyapunov functions. Numerical simulations were conducted to confirm our analytic results. Our findings was that, reducing the contact rate between the non-drinkers and heavy drinkers, increasing the number of drinkers that go into treatment and educating drinkers to refrain from drinking can be useful in combating the drinking epidemic.

Keywords: Equilibrium points; drinking free equilibrium; endemic equilibrium; reproductive number; global stability; Lyapunov function.

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1 Introduction

Alcohol is the world's third leading cause of ill health and premature death, after low birth weight and unsafe sex. In Europe, alcohol is also the third leading risk factor for disease and mortality after tobacco and high blood pressure [1,2]. Excessive drinking is not only harmful to personal health, but also leads to a range of negative social effects such as violence, antisocial and criminal behavior. Long-term alcoholism will produce negative change in the brain, such as intolerance and poor physical dependence. Alcohol damages almost all parts of the body and contributes to a number of human disease including liver cirrhosis, pancreatitis, heart disease, sexual dysfunction, and eventually death [3]. For most individuals, the development of the colorectal cancer (CRC) is sporadic, and one of the risk factors include excessive alcohol intake [4]. Damage to the central and peripheral nervous systems can occur from sustained alcohol consumption [5-7]. Additionally, heavy drinking women have been found to have a negative effect on the reproductive functioning [8,7]. The World Health Organization reports that the harmful use of alcohol causes approximately 3.3 million deaths every year (or 5.9% of all the global deaths), and 5.1% of the global burden of disease is attributed to alcohol consumption. Although there had been many attempts to reduce the problem, alcohol by young people had persisted and in some cases increased over the past several years [3,9].

Mathematical models could mimic the process of drinking and provide useful tools to analyze the spread and control of drinking behavior. Several different mathematical models for drinking had been formulated and studied [10]. In 2008, [11] proposed a mathematical model to study the dynamics of campus drinking as an epidemiological model. They divided their population into three classes: non-drinkers (N), social drinkers (S) and problem drinkers (P). According to their results, the reproductive numbers were not sufficient to predict whether drinking behavior would persist on campus and that the pattern of recruiting new members played a significant role in the reduction of campus alcohol problems. Another model which is relevant to this research is that of [12]. They presented a model to investigate the global property of a drinking model with public health educational campaigns. They divided their population into five compartments namely susceptible drinkers (S), educational drinkers (E), alcoholic drinkers (A), temporary removed drinkers (R), and quit drinkers (Q). They derived the global stability and the basic reproductive number of their model. Their result indicated that the public health educational campaigns of drinking individuals can slow down the drinking dynamics. Also, Xian et al. [10] presented a quit drinking model taking into account permanent quit drinker's compartment and relapse, global stability of equilibria was obtained.

In this paper, we use a modified SDTRS model of [13] to model alcoholism as epidemic. In their model, it was possible for a heavy drinker in treatment to become a heavy drinker without passing through the recovery compartment. In this paper, we assume that heavy drinkers in treatment will only become heavy drinkers again after passing through the recovery and susceptible compartments respectively. The paper is organized as follows: In section 2, we present the model description and assumptions. Stability analysis of the drinking- free and endemic equilibria is discussed in section 3. In Section 4, we use the numerical example to show the dynamical behaviour of our results. In Section 5, we performed the sensitivity analysis of the basic reproductive number of the model. Section 6 is made up of discussion of our results. We ended the paper with a conclusion.

2 Mathematical Model

2.1 Model description

We formulate a mathematical model and divide the population into four compartments: non- drinkers (S), heavy drinkers (H), drinkers in treatment (T), and recovered drinkers (R). The interaction between the four drinking states are shown in the schematic diagram in Fig. 1.

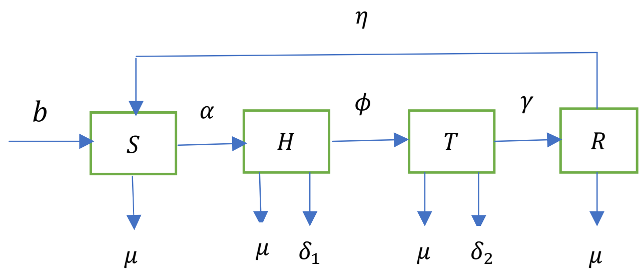

Fig. 1. Schematic diagram of the four drinking classes in the model

2.2 Model assumptions

The following assumptions were made in the model:

(i) The drinking epidemic occurs in a closed environment.
(ii) Sex, race and social status do not affect the probability of becoming a heavy drinker.
(iii) Heavy drinking is transmitted to non- drinkers when they are in contact with heavy drinkers
(iv) Members mix homogeneously (have the same interaction to the same degree)
(v) Drinkers in treatment may only become heavy drinkers again after passing through the recovery and susceptible compartments respectively and
(vi) Drinkers who have stopped drinking enter into recovery compartment.

The drinking epidemic is modelled using the system of nonlinear Differential Equations.

$$\frac{dS}{dt} = b - \alpha SH - \mu S + \eta R \tag{1}$$

$$\frac{dH}{dt} = \alpha SH - (\mu + \delta_1 + \phi)H \tag{2}$$

$$\frac{dT}{dt} = \phi H - (\mu + \delta_2 + \gamma)T \tag{3}$$

$$\frac{dR}{dt} = \gamma T - (\mu + \eta)R \tag{4}$$

with the initial conditions $S(0) \geq 0$, $H(0) \geq 0$, $T(0) \geq 0$ and $R(0) \geq 0$, where

b= recruitment rate of S
α=transmission rate from S to H
η=transmission rate from R to S
μ=natural death rate
δ_1=drinking induced death rate of H
δ_2= drinking induced death rate of T
ϕ= proportion of drinkers entering T compartment and
γ= recovered rate of T

We assume that the system of nonlinear differential equations $(1) - (4)$ has positive initial conditions, then every solution $(S(t), H(t), T(t), R(t))$ of $(1) - (4)$ has the positive properties, that is, $S(t) \geq 0$, $H(t) \geq 0, T(t) \geq 0$ and $R(t) \geq 0$. Hence the feasible region $\Gamma = \{(S, H, T, R)\epsilon R_+^4 : S + H + T + R \leq \frac{b}{\mu}\}$, is positively invariant set for the system $(1) - (4)$.

This implies that:

$$N(t) = S(t) + H(t) + T(t) + R(t) \tag{5}$$

also

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dH}{dt} + \frac{dT}{dt} + \frac{dR}{dt}$$

$$\frac{dN}{dt} = b - \mu N - (\delta_1 H + \delta_2 T) \leq b - \mu N \tag{6}$$

From (6), it follows that:

$$\lim_{t\to\infty} Sup\, N(t) \leq \frac{b}{\mu}.$$

Thus, the feasible region of the system $(1) - (4)$ is given by the set Γ.

3 Model Analysis

3.1 Drinking-free equilibrium and basic reproductive number

In this section, we consider the drinking -free equilibrium $E_0 = (\frac{b}{\mu}, 0, 0, 0)$. That is a situation where there is no drinking problem. We analyze the stability of the drinking-free equilibrium by considering the linearized system of ODE's $(1) - (4)$, taking the Jacobian matrix and obtained

$$J(S,H,T,R) = \begin{bmatrix} -(\alpha H + \mu) & -\alpha S & 0 & \eta \\ \alpha H & \alpha S - (\mu + \delta_1 + \phi) & 0 & 0 \\ 0 & \phi & -(\mu + \delta_2 + \gamma) & 0 \\ 0 & 0 & \gamma & -(\mu + \eta) \end{bmatrix} \tag{7}$$

The local stability of the equilibrium may be determined from the Jacobian matrix (7). This implies that the Jacobian matrix for the drinking-free equilibrium is given by

$$J(E_0) = \begin{bmatrix} -\mu & -\alpha & 0 & \eta \\ 0 & \alpha - (\mu + \delta_1 + \phi) & 0 & 0 \\ 0 & \phi & -(\mu + \delta_2 + \gamma) & 0 \\ 0 & 0 & \gamma & -(\mu + \eta) \end{bmatrix} \tag{8}$$

From the characteristic equation of $J(S, 0, 0, 0)$, the following eigenvalues were obtained: $\lambda_1 = -\mu$,

$\lambda_2 = \alpha - (\mu + \delta_1 + \phi), \lambda_3 = -(\mu + \delta_2 + \gamma)$ and $\lambda_4 = -(\mu + \eta)$. It can be seen that $\lambda_1, \lambda_3, \lambda_4$ are real and negative. We know that $R_0 < 1$, this implies that $\alpha < (\mu + \delta_1 + \phi)$ and hence λ_2 is therefore real and negative. This means that the system $(1) - (4)$ is asymptotically stable.

The basic reproductive number (R_0), is given by

$$R_0 = \frac{\alpha}{(\mu + \delta_1 + \phi)} \tag{9}$$

Theorem 1: The drinking-free equilibrium $E_0\left(\frac{b}{\mu}, 0, 0, 0\right)$ of the system $(1) - (4)$ is asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

3.2 Endemic equilibrium

We evaluate the equilibrium points of the ODE $(1) - (4)$ by setting the right -hand side of equation $(1) - (4)$ to zero and then solve for S^*, H^*, $T^*$and R^*. We obtained:

$$S^* = \frac{\mu+\delta_1+\emptyset}{\alpha},\ H^* = \frac{\mu(\mu+\delta_1+\emptyset)-\alpha b}{\alpha[\eta-(\mu+\delta_1+\emptyset)]}, T^* = \frac{\emptyset[\mu(\mu+\delta_1+\emptyset)-\alpha b]}{\alpha(\mu+\delta_2+\gamma)[\eta-(\mu+\delta_1+\emptyset]},$$

and

$$R^* = \frac{\gamma\emptyset[\mu(\mu+\delta_1+\emptyset)-\alpha b]}{\alpha(\mu+\eta)(\mu+\delta_2+\gamma)[\eta-(\mu+\delta_1+\emptyset]}. \tag{10}$$

We now consider the case when $R_0 > 1$. At the endemic equilibrium, all the four drinking states are present in the population. The steady states consider conditions under which all four drinking states can coexist in the equilibrium. We represent $E^* = \left(S^*\ ,\ H^*\ ,\ T^*\ ,\ R^*\right)$ as endemic equilibrium of the system $(1) - (4)$ and $\left(S^* \neq 0\ ,\ H^* \neq 0\ ,\ T^* \neq 0\ ,\ R^* \neq 0\right)$. We Substitute the equilibrium points in (10) into equation (7), and obtain

$$J(E^*) = \begin{bmatrix} -\left[\alpha\left[\frac{\mu(\mu+\delta_1+\emptyset)-\alpha b}{\alpha[\eta-(\mu+\delta_1+\emptyset)]}\right]+\mu\right] & -\alpha\left[\frac{\mu+\delta_1+\emptyset}{\alpha}\right] & 0 & \eta \\ \alpha\left[\frac{\mu(\mu+\delta_1+\emptyset)-\alpha b}{\alpha[\eta-(\mu+\delta_1+\emptyset)]}\right] & \alpha\left[\frac{\mu+\delta_1+\emptyset}{\alpha}\right]-(\mu+\delta_1+\phi) & 0 & 0 \\ 0 & \phi & -(\mu+\delta_2+\gamma) & 0 \\ 0 & 0 & \gamma & -(\mu+\eta) \end{bmatrix} \tag{11}$$

Let

$$A_{11} = -\left[\alpha\left[\frac{\mu(\mu+\delta_1+\emptyset)-\alpha b}{\alpha[\eta-(\mu+\delta_1+\emptyset)]}\right]\right]-\mu, A_{12} = -\alpha\left[\frac{\mu+\delta_1+\emptyset}{\alpha}\right],\ A_{13} = 0, A_{14} = \eta,$$

$$A_{21} = \alpha\left[\frac{\mu(\mu+\delta_1+\emptyset)-\alpha b}{\alpha[\eta-(\mu+\delta_1+\emptyset)]}\right],\ A_{22} = \alpha\left[\frac{\mu+\delta_1+\emptyset}{\alpha}\right]-(\mu+\delta_1+\phi), A_{23} = 0,\ A_{24} = 0,$$

$$A_{31} = 0, A_{32} = \phi, A_{33} = -(\mu+\delta_2+\gamma), A_{34} = 0$$

$$A_{41} = 0, A_{42} = 0, A_{43} = \gamma, A_{44} = -(\mu+\eta)$$

Substituting A_{ij} into (12), we obtain

$$J(E^*) = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{33} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{41} & A_{44} \end{bmatrix} \tag{12}$$

The characteristic equation of (13) can be computed as follows

$$|J(E^*) - \lambda I| = \begin{vmatrix} A_{11}-\lambda & A_{12} & 0 & A_{14} \\ A_{21} & A_{22}-\lambda & 0 & 0 \\ 0 & A_{32} & A_{33}-\lambda & 0 \\ 0 & 0 & A_{43} & A_{44}-\lambda \end{vmatrix} = 0$$

$$\lambda^4 + (A_{11}+A_{22}+A_{33}+A_{44})\lambda^3 + (A_{12}A_{21} - A_{33}A_{44} - A_{11}A_{44} - A_{22}A_{44} - A_{11}A_{33} - A_{22}A_{33} \\ -A_{11}A_{22})\lambda^2 + (A_{11}A_{33}A_{44} + A_{22}A_{33}A_{44} + A_{11}A_{22}A_{44} + A_{11}A_{22}A_{33} - A_{12}A_{21}A_{33} \\ -A_{12}A_{21}A_{44})\lambda + (A_{12}A_{21}A_{33}A_{44} + A_{14}A_{21}A_{32}A_{43} - A_{11}A_{22}A_{33}A_{44}) = 0$$

We can write the characteristic equation above as:

$$\lambda^4 + C_1\lambda^3 + C_2\lambda^2 + C_3\lambda + C_4 = 0 \tag{13}$$

Where:

$$C_1 = A_{11} + A_{22} + A_{33} + A_{44}$$
$$C_2 = A_{12}A_{21} - A_{33}A_{44} - A_{11}A_{44} - A_{22}A_{44} - A_{11}A_{33} - A_{22}A_{33} - A_{11}A_{22}$$
$$C_3 = A_{11}A_{33}A_{44} + A_{22}A_{33}A_{44} + A_{11}A_{22}A_{44} + A_{11}A_{22}A_{33} - A_{12}A_{21}A_{33} - A_{12}A_{21}A_{44}$$
$$C_4 = A_{12}A_{21}A_{33}A_{44} + A_{14}A_{21}A_{32}A_{43} - A_{11}A_{22}A_{33}A_{44}$$

Using the Routh –Hurwitz criterion [13,14]. It can be seen that all eigenvalues of the characteristic equation (13) has negative real part if and only if:

$$C_1 > 0, C_4 > 0\,, C_1C_2 - C_3 > 0\,, (C_1C_2 - C_3)C_3 - {C_1}^2C_4 > 0 \tag{14}$$

Theorem 2: E^* is asymptotically stable if and only if inequalities (14) is satisfied.

3.3 Global stability of the equilibrium points

3.3.1 Global stability of the drinking free equilibrium

We prove the global stability when $\alpha \leq \mu$.

Theorem 3: The global stability E_0 is asymptotically stable in the region

$\Gamma = \left\{(S,H,T,R) \in R_+^4 : S + H + T + R \leq \frac{b}{\mu}\right\}$ if $\alpha \leq \mu$ (note that $\alpha \leq \mu$ implies $R_0 < 1$).

Proof: It should be noted that $S < 1$ in Γ for time $(t) > 1$. Consider the Lyaponov function:

$$L = H + T$$
$$\frac{dL}{dt} = (\alpha S - \mu - \delta_1)H - (\mu - \delta_2 + \gamma)T$$
$$\leq (\alpha - \mu - \delta_1)H - (\mu - \delta_2)T$$

$\frac{dL}{dt} < 0$ for $\alpha \leq \mu$ and $\frac{dL}{dt} = 0$ only if $H = 0$ and $T = 0$. Therefore, the only trajectory of

the system in which $\frac{dL}{dt} = 0$ is E_0. Hence, Lasalle's invariance principle, E_0 is globally asymptotically stable in Γ [15-17,14].

3.3.2 Global stability of the endemic equilibrium ($\mathbf{E}^*$)

We determine the global stability of the endemic equilibrium in this section, by using the first three equations of the system $(1) - (4)$ that is:

$$\begin{aligned} \frac{dS}{dt} &= b - \alpha SH - \mu S + \eta R \\ \frac{dH}{dt} &= \alpha SH - (\mu + \delta_1 + \phi)H \\ \frac{dT}{dt} &= \phi H - (\mu + \delta_2 + \gamma)T \end{aligned} \tag{15}$$

in the region $\Gamma^* = \{(S,H,T)\epsilon R_+^3 : S + H + T \leq 1, S > 0\,, H \geq 0\,, T \geq 0\}, \Gamma^*$ is positively invariant, i.e. every solution of the model (15), with initial conditions in Γ^* remains there for time $(t > 0)$.

We also consider

$\Gamma^{**} = \left\{(S,H,T): S + \left(\frac{\mu+\delta_1}{\mu}\right)H + \left(\frac{\mu+\delta_2+\gamma}{\mu}\right)T = 1, S > 0\ , H \geq 0\ , T \geq 0\right\}$ where $\Gamma^{**} \subset \Gamma^{*}, \Gamma^{**}$ is positively invariant, $E^{*} \in \Gamma^{*}$ and $b = \mu$.

Theorem 4: The endemic equilibrium point E^{*} of model (15) is globally asymptotically stable if $R_0 > 1$(This means that $\emptyset \leq \alpha$).

Proof: From **theorem 1**, if $R_0 > 1$ in Γ^{**}, then E_0 is unstable. Also Γ^{**} is positively invariant subset of Γ^{*} and the ω-limit set of each solution of model (14) is a single point in Γ^{**} since there is no periodic solutions, homoclinic loops and oriented phase polygons inside Γ^{**} if $\emptyset \leq \alpha$. Therefore E^{*} is globally asymptotically stable [15,16].

4 Numerical Example

In this section, we use numerical simulations to show the dynamical behaviour of our results, by assuming that our total population is 100% and choose $S = 0.50, H = 0.25$, $T = 0.15$ and $R = 0.1$. The other parameters that would be used in this section are displayed in Table 1 and Table 2 respectively.

Table 1. Model parameters at drinking free equilibrium

Parameter	Description	Value	Source
b	Recruitment rate of S	0.4	[18]
α	Transmission rate from S to H	0.7	[18]
η	Transmission rate from R to S	0.1	[18]
μ	Natural death rate	0.25	[18]
δ_1	Drinking induced death rate of H	0.35	[18]
δ_2	Drinking induced death rate of T	0.3	[18]
γ	Recovered rate of T	0.09	[18]
ϕ	Proportion of drinkers entering T compartment	0.7	[18]

Table 2. Model parameters at endemic equilibrium

Parameter	Description	Value	Source
b	Recruitment rate of S	0.4	[18]
α	Transmission rate from S to H	0.7	[18]
η	Transmission rate from R to S	0.01	[18]
μ	Natural death rate	0.025	[18]
δ_1	Drinking induced death rate of H	0.035	[18]
δ_2	Drinking induced death rate of T	0.03	[18]
γ	Recovered rate of T	0.1	[18]
ϕ	Proportion of drinkers entering T compartment	0.5	[18]

5 Sensitivity Analysis of the Basic Reproductive Numbers

We investigate the nature of the model by conducting sensitivity analysis of the reproductive number (R_0).

(a) At the drinking –free equilibrium, $\alpha = 0.7$, $\delta_1 = 0.35, \emptyset = 0.7$ and $\mu = 0.25$, $R_0 = 0.5385 < 1$.

(i) If the value of α is increased to any figure greater than 1.31 and the values of $\emptyset$, μ, δ_1 are maintained $R_0 > 1$.
(ii) If the value of $\emptyset$ is reduced to 0.068 and the values of α, δ_1 and μ are maintained the same, $R_0 > 1$.

(b) At the endemic equilibrium, $\alpha = 0.7$, $\delta_1 = 0.035, \mu = 0.025$, and $\emptyset = 0.5$, $R_0 = 1.25 > 1$.

(i) If α is reduced to 0.5 and $\mu, \delta_1, \emptyset$ are maintained the same, $R_0 < 1$.
(ii) If the values of α, μ and $\delta_1$are maintained at 0.7 and 0.025 and 0.035 and $\emptyset$ is increased to 0.8, $R_0 < 1$.

6 Discussion of Results

We use $SHTR$ model to study the dynamics of drinking as an epidemic. We discussed the existence and stability of drinking free and endemic equilibria and performed the sensitivity analysis of the reproductive numbers. Based on the data in Table 1, the basic reproductive number of the drinking free equilibrium is estimated to be $R_0 = 0.5385 < 1$. This implies that only non-drinkers population is present and the heavy drinkers, drinkers in treatment and recovered drinkers' populations reduces to zero ($H = 0, T = 0, R = 0$). This means that the model is asymptotically stable at $R_0 < 1$ and satisfies Theorem1. This has been verified numerically in Fig. 2. In the stability analysis of the drinking free equilibrium, the eigenvalues are $\lambda_1 = -0.25$, $\lambda_2 = -0.6$, $\lambda_3 = -0.64$ and $\lambda_4 = -0.35$. This also indicates that the drinking free equilibrium is asymptotically stable.

Considering the situation when $R_0 > 1$, the reproductive number of the endemic equilibrium is estimated to be $R_0 = 1.25 > 1$ using the data in Table 2. This shows the situation in which the non-drinkers, heavy drinkers, drinkers in treatment and recovered drinkers coexist in the population $(S^*, H^*, T^*, R^*) = (0.8, 0.6909, 2.2287, 6.3678)$]. This indicates the existence of drinking problem in the population. People with drinking problem will continue to transform more non-drinkers into heavy drinkers and the drinking free equilibrium becomes unstable at $R_0 > 1$. This is in line with our analytical results and has also been verified numerically in Fig. 3.

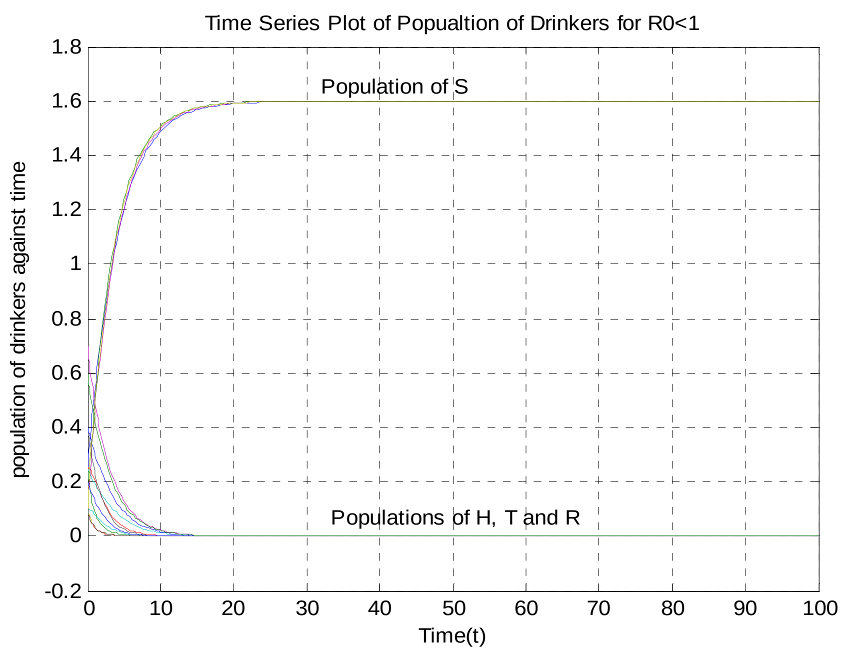

Fig. 2. When $R_0 = 0.5385$, only non-drinkers exist. The populations of heavy drinkers, drinkers in treatment and recovered drinkers, approach zero and reach disease free equilibrium

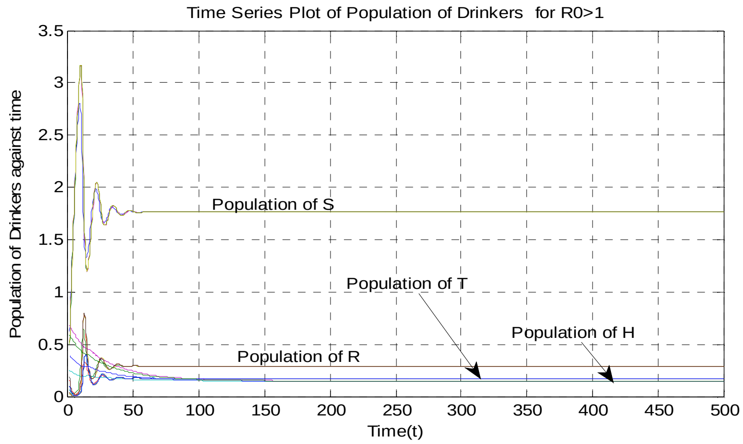

Fig. 3. When $R_0 = 1.25$, all the distinct drinking classes coexist and therefore approach endemic equilibrium

At the sensitivity analysis of the basic reproductive number of the endemic equilibrium, if α is reduced from 0.7 to 0.5 and $\mu, \delta_1, \emptyset$ maintained the same, $R_0 < 1$. Furthermore, if $\emptyset$ is increased from 0.5 to 0.8, $R_0 < 1$. Also$R_0 > 1$ at the drinking -free equilibrium, if either α is increased to any figure greater than 1.31 or $\emptyset$ is reduced below 0.1.

7 Conclusion

Our model shows that, drinking epidemic cannot only be controlled by reducing the contact rate between the non-drinkers and heavy drinkers but also increasing the number of drinkers that go into treatment and educating drinkers to refrain from drinking can be useful in combating the epidemic.

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Competing Interests

Authors have declared that no competing interests exist.

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