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Seismic Raypath and Wavefront Models in the Lithofacies of the Niger Delta, Nigeria

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

Seismic raypaths and wavefronts model in the vertically anisotropic lithofacies of the Niger Delta are presented. These models based on the linear increase of velocity with depth are defined by the parametric equations: For raypaths [$x(\theta) = R_r \text{Cos}\ \theta + x_{rc}$ (centre) and $z(\theta) = R_r\, Sin\ \theta - z_{rc}$ (radii)]; and for Wavefronts [$x(\theta) = R_w\, Cos\ \theta$ (centre) and $z(\theta) = R_w\, Sin\ \theta + z_w$ (radii)]. The take-off velocity, V_0 = 1656 ms^{-1} and vertical velocity gradient, k = 0.44 s^{-1} previously determined for the Niger Delta served as input to these equations to generate the models in *Graph (version 4.3)* software. Raypaths are at offsets from 400 to 2000 m at 400 m intervals while the wavefronts are at travel times from 20 to 100 ms at 20 ms intervals. Raypaths are observed to be different from straight lines and travel in circular paths while wavefronts are circular arcs travelling greater distance in the vertical direction than in the horizontal for equal travel times. These models provide a basic framework in the Niger Delta for accurate ray tracing, velocity models development and traveltime calculations in seismic processing.

Keywords: Raypaths; wavefronts; lithofacies; linear velocity function and Niger Delta.

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1. INTRODUCTION

In recent years, there has been an increase in the development of new methods for predicting source-receiver paths taken by seismic waves in the presence of lateral variations in wavespeed. Ray tracing had been carried out using the ordinal wavefront reconstruction method [1] while ray series method and dynamic ray-tracing system was developed for 3-D inhomogeneous media [2]. [3] did ray tracing by wavefront construction for anisotropic media, [4] conducted seismic ray tracing and wavefront tracking in laterally heterogeneous media and [5] estimated traveltime and amplitude using wavefront construction. In all of these methods, the exploitation of multi-arrivals of raypaths and wavefronts resulted in improved images.

Seismic raypath and wavefront models have been generally based on the assumption of a linear relationship between velocity and path geometry which make them simpler to handle and have usually led to sufficient accuracy for relatively shallow homogenous and isotropic horizons exhibiting gentle relief where the seismic velocity laterally or vertically remains constant. This constant velocity theory which assumes straight raypaths and wavefronts for a realistic earth model is inappropriate for areas where there are dramatic changes of velocity, such as thick carbonate or evaporate units alternating with thick elastic units (as those found in the southern North Sea Basin), complex structures, tectonic inversions or lateral lithology change and clastic sedimentary basins of the world such as the Gulf Coast of the United States and Niger Delta of Nigeria.

The assumption of straight raypaths and wavefronts are not applicable with the observed variation of velocity with depth in the Niger Delta [6]. Research has shown that a linear variation of velocity with depth can be allowed for but the curvature of the path produced by this variation is ignored [7]. Such a method succeeds in reconciling the inconsistency mentioned above but unfortunately in many cases it still exposes itself to possibly a greater error by ignoring an appreciable deviation of the velocity gradient from the vertical which in general accompanies strongly dipping strata.

Considering the inconsistencies with the linear variation of velocity with depth and straight raypath and wavefront geometries in homogeneous medium, this paper presents a model to minimize these inconsistencies by the use of velocity functions with an earth model to construct raypaths and wavefronts applicable to the homogeneous and anisotropic lithofacies of the Niger Delta.

2. LITHOSTRATIGRAPHY OF THE NIGER DELTA

As in many deltaic areas, it is extremely difficult to define a satisfactory stratigraphic nomenclature [8]. Within these heterogeneous formations are intercalations of thin clastic lithofacies: conglomerates, sandstones, siltstones and shales. These intercalations make it difficult to define units and formation

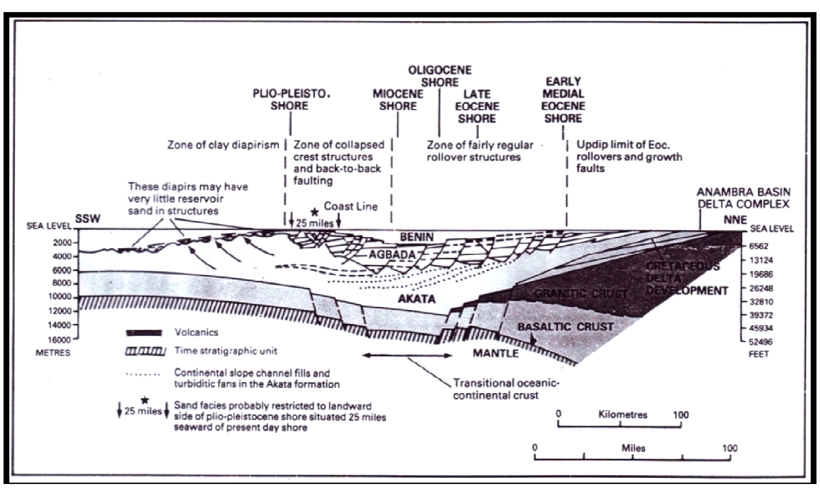

Fig. 1. The different lithofacies in the Niger Delta (based on [11] and [12] and modified by [13])

boundaries as homogeneous, isotropic, discrete and sharply discontinuous layers with sufficient integrity. However, three Formation names are in wide-spread use [9] and [10], corresponding to the portions of the tripartite sequence (Fig. 1).

The first is known as the Akata Formation [9]. This is composed of the following lithofacies: marine shales, clays and silts at the base of the known delta sequence. They contain a few streaks of sand, possibly of turbidite origin, and were deposited in delta-front to deeper marine environments. The thickness of this sequence is not known but may reach 7000 m in the central part of the delta. Marine shales form the base of the sequence in each depobelt and range from Paleocene to Holocene in age. They crop out offshore in diapirs along the continental slope, and onshore in the North-Eastern part of the delta, where they are known as the Imo Shale. Except on the basin flanks, no wells have fully penetrated this sequence. The Akata Formation is typically overpressured.

Overlying the Akata Formation is the Agbada Formation [9] or the paralic clastics. This forms the hydrocarbon-prospective sequence in the Niger Delta. It is represented by an alternation of sands, silts and clays in various proportions and thicknesses, representing cyclic sequences of offlap units. The Agbada Formation was deposited in a number of delta-front, delta-topset, and fluvio-deltaic environments. The alternation of fine and coarse clastics provides multiple reservoir-seal couplets. As with the Akata Formation, the Agbada Formation is present in all depobelts, and ranges in age from Eocene to Pleistocene. Most exploration wells in the Niger Delta have bottomed in this Agbada Formation, which reaches a maximum thickness of more than 3000m. This formation has its outcrops at Ogwashi, Asaba and Ameki.

Following the Agbada Formation [9] is the overlying Benin Formation also known as the Continental Sand. The shallowest part of this sequence is composed almost entirely of non-marine sands. It was deposited in alluvial or upper coastal plain environment following a southward shift of deltaic deposition into a new depobelt. The oldest continental sands are probably Oligecene, although they lack fauna and are impossible to date directly. Offshore they become thinner and disappear near the shelf edge. The present outcrops of this formation could be seen around Owerri, Benin and Onitsha.

3. FUNDAMENTAL MODEL EQUATIONS

The fundamental model equations for a medium (Fig. 2) in which the velocity, V is a function only of the depth for a given ray are expressed in parametric forms by [14]:

$$x = \int_0^H \frac{pV(z)dz}{\left[1 - p^2V^2(z)\right]^{1/2}} \tag{1}$$

$$t = \int_0^H \frac{dz}{V(z)\left[1 - p^2V^2(z)\right]^{1/2}} \tag{2}$$

$$p = \frac{\sin \alpha}{v} \tag{3}$$

Where:

x = horizontal displacement of the ray
t = travel time along the ray
H = thickness of the layer
α = angle between the ray and the vertical at any point
p = ray parameter or slowness

With the ray parameter *p* being fixed, equation (1) represents the equations of the raypath from the source point in the (x, z) plane.

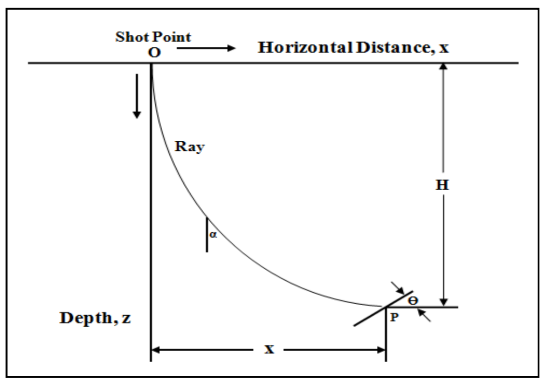

Fig. 2. Path of normal ray from shot-point to reflecting horizon

3.1 Raypath Equations

The solution to the integral equations (1) and (2) gives the linear with depth velocity function:

$$V(z) = V_0 + kz \tag{4}$$

The general expressions for the (x, z) coordinates of the ray path for a source point located at (x_0, z_0) are given by [15] as:

$$x = \left(z_o + \frac{V_0}{k} \frac{Cosi_0 - Cosi}{Sini_0} \right) + x_0 \tag{5}$$

$$z = \frac{Sini}{Sini_0} \left(z_0 + \frac{V_0}{k} \right) - \frac{V_o}{k} \tag{6}$$

For the simple case of $x_0 = 0$, $z_0 = 0$, equations (5) & (6) take the form:

$$x = \frac{1}{pk}\left(Cosi_0 - Cosi\right) \tag{7}$$

$$x = \frac{1}{pk}\left(Sini_0 - Sini\right) \tag{8}$$

The angle of emergence i_0 can be obtained from the equation:

$$x_i = 2\left(\frac{V_0}{k}\right)Coti_0 \tag{9}$$

Where x_i = The horizontal surface distance where the ray emerges.

A physical ray can only exist on the half circle described by equations (7) and (8) (or below) the line $z = -\frac{V_0}{k}$ where the velocity is positive. The velocity on the other half circle is negative, where a physical ray does not exist [15]. Since the ray paths are circular, the curvature denoted by Γ will be constant.

$$\Gamma = \left|\frac{di}{ds}\right| = \frac{kSini}{V(z)} = \frac{kSini_0}{V(z_0)} = \text{Constant} \tag{10}$$

Where s is the ray path length. And the radius of the ray path is given by:

$$R_r = \frac{1}{\Gamma} = \frac{V(z)}{kSini} = \frac{V(z_0)}{kSini_0} = \text{Constant} \tag{11}$$

$$R_r = \frac{1}{pk} \tag{12}$$

The x and z components of the raypath centres are given by:

$$x_{rc} = \frac{1}{pk}\left(1 - p^2 V_0^{\ 2}\right)^{1/2} \tag{13}$$

$$z_{rc} = -\frac{V_0}{k} \tag{14}$$

3.2 Wavefront Equations

The equations for the depth, z_{wc} to the centre of a wavefront circle below the surface and the Radius of curvature R_w are given by [16]:

$$Z_{wc} = \frac{V_0}{k}\left(Cosh[kt] - 1\right) \tag{15}$$

$$R_w = \frac{V_0}{k} Sinhkt \tag{16}$$

Both equations show that z_{wc} and R_w are dependent on the one-way travel time t.

Equation (15) can also be expressed in the form:

$$z_{wc} = \frac{1}{2}\frac{x_i^{\ 2}k}{V_0} \tag{17}$$

Equation (16) gives the instantaneous depth of the centre of the wavefront in terms of the horizontal distance x_i at which the wavefront intersects the surface. Equation (16) can be also expressed as:

$$R_w = \frac{x_i}{2V_0}\left(x_i^{\ 2}k^2 + 4V_0^{\ 2}\right)^{1/2} \tag{18}$$

The expressions of equations (17) and (18) are of more interest in refraction than in reflection work [16].

4. FIELD DATA AND LOCATION

The study area is the Agbada field located in OML 17 approximately 16 km northeast of Port Harcourt. The Survey is situated between Latitude 4°55' and 5°10' north and between longitude 6°50' and 7°10' east in the Central Niger Delta of Nigeria (Fig. 3).

The Time-Offset (t-x) data used in this study were extracted from a 3-D Seismic Reflection Survey conducted within the Central Niger Delta. The prospect consisted of a regular grid configuration of 73 north-by-south running receiver lines increasing by 5 and 60 west-by-east source lines increasing by 8 (Fig. 4). Receiver spread of 480 channels divided into six separate lines of 80 stations each was used throughout the program. Receiver and source lines were spaced at 250 and 400 m respectively. Geophones and source pegs were evenly spaced at 50 m. A 15- fold, non-symmetric split-spread geophones and shots were covered. The explosive energy source comprising 0.2 kg dynamite buried in 5 or 10 pattern holes each 3.0 or 6.0 m deep was used.

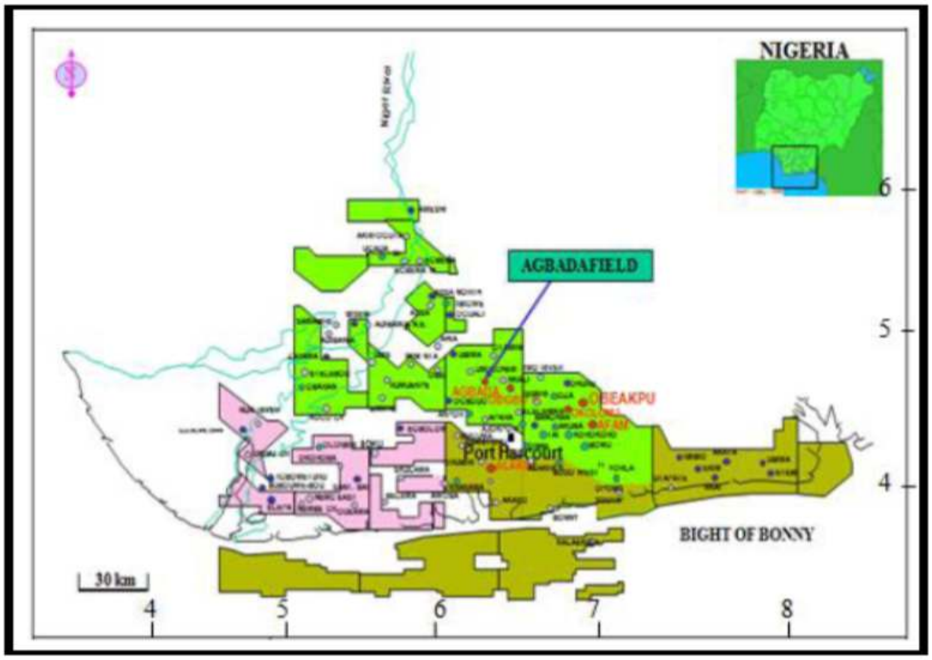

Fig. 3. Map showing the agbada Field of the Central Niger Delta Basin

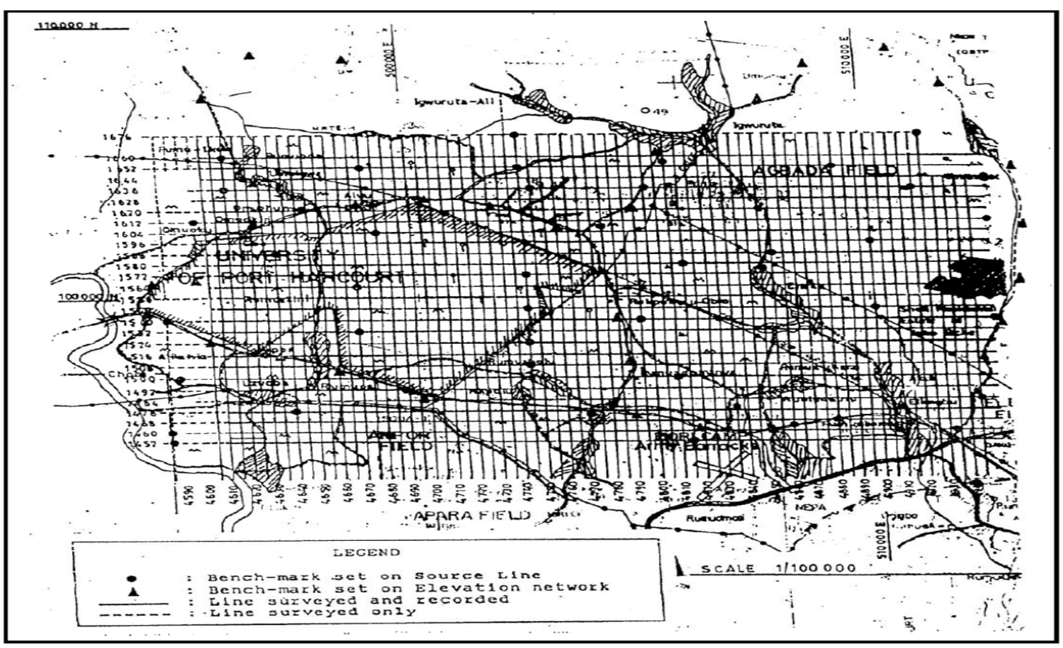

Fig. 4. Agbada 3-D program map showing receiver and source line

5. MODELLING APPROACH

The geometrical models are guided by assumptions that: raypaths are circular arcs within each layer with centres that lie on a horizontal line a distance V_0/k above the x-axis; wavefronts are also circular arcs with centres at depths Z_{wc} along the z-axis with radii R_w and layers are horizontal. The seismic waves are modelled to propagate from a source point (located on the surface) with take-off velocity V_0 overlying a half space through a medium with velocity gradient, k, consisting of a top layer.

5.1 Velocity Gradient (*k*)

The required velocity gradient, k was obtained from a velocity model study by [6] in the Niger Delta as: $k = 0.44\ s^{-1}$

5.2 Take off Velocity (V_0) and Layer Thickness (h_1)

The take-off velocity was similarly obtained by [6] as: $V_0 = 1656\ ms^{-1}$; and the thickness of the layer for the model is assumed to be $h_1 = 80$ m.

5.3 Ray Parameter, p

Snell's law was used to obtain the expression for the ray parameter for each ray:

$$\frac{Sini}{V(z)} = \frac{Sini_0}{V_0} = p \tag{19}$$

Where i is the angle the ray makes with the vertical at any point and V(z) is the velocity (as a function of depth) at that point, p is a ray parameter that remains constant for any particular ray, and hence distinguishes one ray from another [7].

6. MODEL CONFIGURATION

6.1 Earth Model Configuration

The typical earth model in which the wave is modeled to propagate is shown in Fig. 5. The horizontal at $z = h_0$ (where $h_0 = 0$ represents the surface) is the top of the 1st layer and the horizontal at $z = h_1$ represents the boundary between the 1st layer and the half space.

The layer $h_0 < z < h_1$ represents the medium with velocity gradient k and thickness $h_0 + h_1$. A 2-dimensional reference coordinate system (x, z) was used to represent the position of a point on the ray at an instant of time; thus the shot point S will be located at $(x_0, z_0) = (0, 0)$. With reference to this coordinate system, the coordinates (x_{rc}, z_{rc}) for the centre of the ray paths are given by:

$$\left[\frac{1}{pk}\left(1 - p^2 V_0{}^2\right)^{1/2}, = \frac{V_0}{k}\right]$$

6.2 Raypath Model Configuration

The raypath model (Fig. 6) depicts the geometry of the seismic rays emanating from the source point located on the surface, travelling through the subsurface and terminating at the surface at a certain distance (offset) x_i from the source point. The emergence angle i_0 and ray parameter p for each ray were determined by equations (9.0) and (19). The raypath centre coordinates and radii where computed using equations (12), (13) and (14) as shown in Table 1.

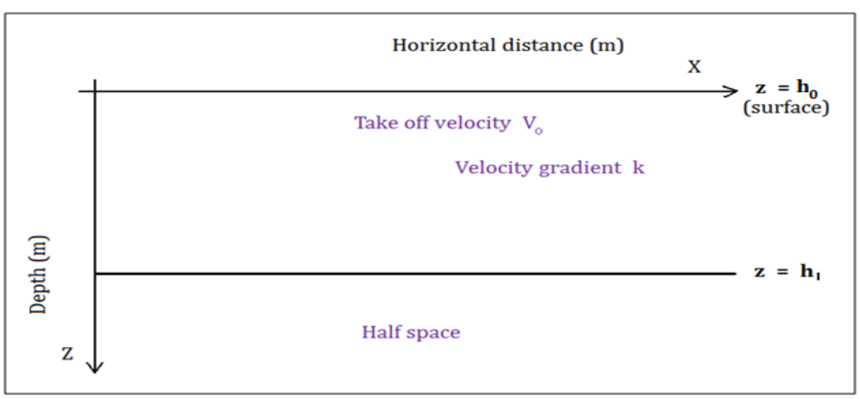

Fig. 5. Earth model configuration with a layer of thickness $h_0 + h_1$ and take off velocity V_0.

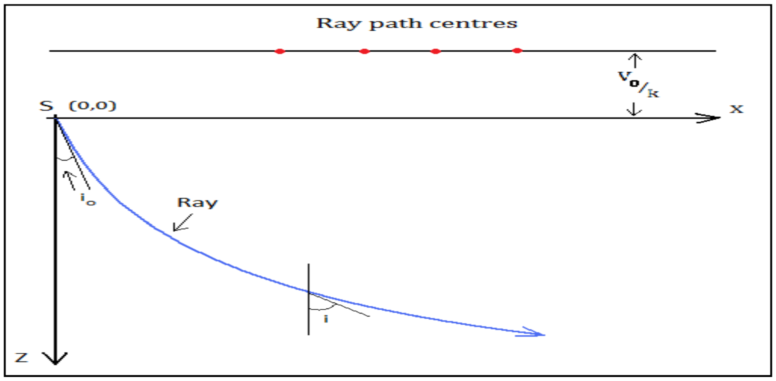

Fig. 6. Ray path trajectory from shot point, S, showing the angle of emergence i_0 and ray path centres

Table 1. Computations for ray path centres and radii for different offsets

X_i (m)	i_0	p	X_{rc} (m)	Z_{rc} (m)	Radius (m)
100	1.5579	0.0005882	50.00	-3863.64	3863.96
200	1.5449	0.0005880	100.00	-3863.64	3864.93
300	1.5320	0.0005878	150.00	-3863.64	3866.55
400	1.5191	0.0005874	200.00	-3863.64	3868.81
500	1.5062	0.0005870	250.00	-3863.64	3871.72
600	1.4933	0.0005865	300.00	-3863.64	3875.27
700	1.4805	0.0005858	350.00	-3863.64	3879.46
800	1.4676	0.0005851	400.00	-3863.64	3884.29
900	1.4548	0.0005843	450.00	-3863.64	3889.75
1000	1.4421	0.0005834	500.00	-3863.64	3895.85
1100	1.4294	0.0005824	550.00	-3863.64	3902.59
1200	1.4167	0.0005813	600.00	-3863.64	3909.95
1300	1.4041	0.0005801	650.00	-3863.64	3917.93
1400	1.3916	0.0005788	700.00	-3863.64	3926.54
1500	1.3791	0.0005775	750.00	-3863.64	3935.76
1600	1.3666	0.0005760	800.00	-3863.64	3945.59
1700	1.3542	0.0005745	850.00	-3863.64	3956.03
1800	1.3419	0.0005729	900.00	-3863.64	3967.08
1900	1.3297	0.0005712	950.00	-3863.64	3978.72
2000	1.3175	0.0005695	1000.00	-3863.64	3990.95
2100	1.3054	0.0005676	1050.00	-3863.64	4003.77
2200	1.2934	0.0005658	1100.00	-3863.64	4017.17
2300	1.2815	0.0005638	1150.00	-3863.64	4031.15
2400	1.2697	0.0005618	1200.00	-3863.64	4045.70
2500	1.2579	0.0005597	1250.00	-3863.64	4060.81

6.3 Wavefront Model Configuration

The wavefront model (Fig. 7) depicts the time evolution of the seismic energy (wavefronts) and describes loci of equal travel time as the wave emanates from the source point and propagates deeper into the subsurface. Equations (15) and (16) were used to express the wavefront centres and radii as functions of one-way travel time t as shown in Table 2.

7. RESULTS AND DISCUSSION

7.1 Construction of Raypaths Model

The raypath centres and radii in Table 1 with the parametric pair equations (20) and (21) as the input equations in *Graph Version 4.3* software, the raypath model is constructed for the following offsets : 400 m, 800 m, 1200 m, 1600 m, 2000 m as shown in Fig. 8.

$$x(\theta) = R_r Cos\theta + x_{rc} \quad (20)$$

$$z(\theta) = R_r Sin\theta - z_{rc} \quad (21)$$

The raypaths (Fig. 8) are circular arcs emanating from the source point. This is a deviation from the conventional straight raypath that is assumed in isotropic layers showing that the linear increase of velocity with depth applies to the earth model in Niger Delta. The raypath model shows direct rays emanating from the source with the rays travelling only within the 1st layer (continuous lines) and others that exceed the 1st layer (dotted lines).

7.2 Construction of Wavefronts Model

The wavefront centres and radii in Table 2 with another set of parametric pair equations (22) and (23) as the input equations in *Graph Version 4.3* software, the wavefront model is constructed for 5 travel times at 20 ms intervals from 20 ms to 100 ms as shown in Fig. 9.

$$x(\theta) = R_w Cos\theta \quad (22)$$

$$z(\theta) = R_w Sin\theta + z_w \quad (23)$$

The wavefronts (Fig. 9) are circular arcs with centres that lie on the z – axis and are located at greater depths as the travel time increases. The wavefronts are concentric circles about the source that increase radially in all directions with time. The 4th wavefront corresponding to 80 ms has travelled a horizontal distance of 136 m while this same wavefront has travelled a vertical distance of 139 m. A similar observation can be made for the last wavefront (100 ms) with a travel distance of 170 m in the horizontal and 175 m in the vertical. This observation reveals the anisotropy of the model and suggests that the wavefront in a medium with a linear increase of velocity with depth does not have the constant curvature that would be obtained from circular wavefronts in an isotropic medium.

Table 2. Computations for wavefront Centres and Radii for different traveltimes

t (ms)	t (s)	Z_{wc} (m)	R_w (m)
10	0.01	0.04	17.00
20	0.02	0.15	34.00
30	0.03	0.34	51.00
40	0.04	0.60	68.00
50	0.05	0.94	85.01
60	0.06	1.35	102.01
70	0.07	1.83	119.02
80	0.08	2.39	136.03
90	0.09	3.03	153.04
100	0.10	3.74	170.05
110	0.11	4.53	187.07
120	0.12	5.39	204.09
130	0.13	6.32	221.12
140	0.14	7.33	238.15
150	0.15	8.42	255.19
160	0.16	9.58	272.22
170	0.17	10.81	289.27
180	0.18	12.12	306.32
190	0.19	13.51	323.38
200	0.20	14.97	340.44

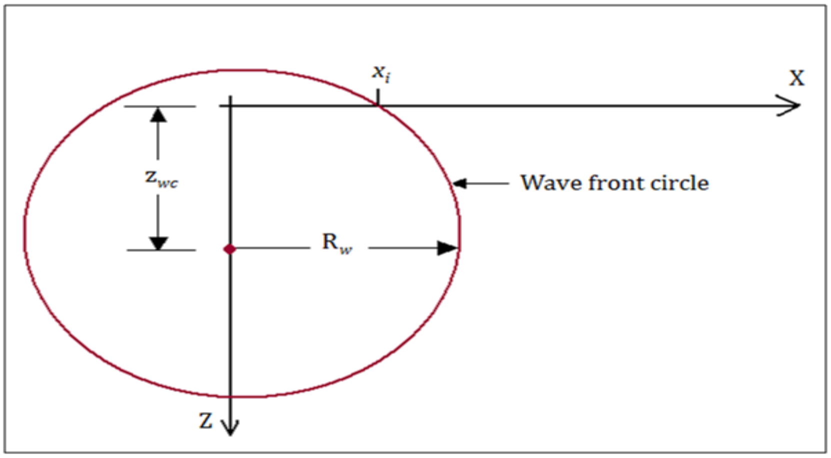

Fig. 7. Wavefront model configuration

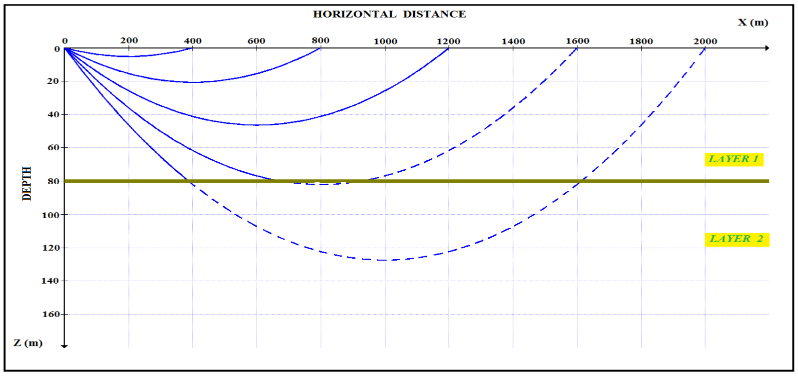

Fig. 8. Constructed raypath model

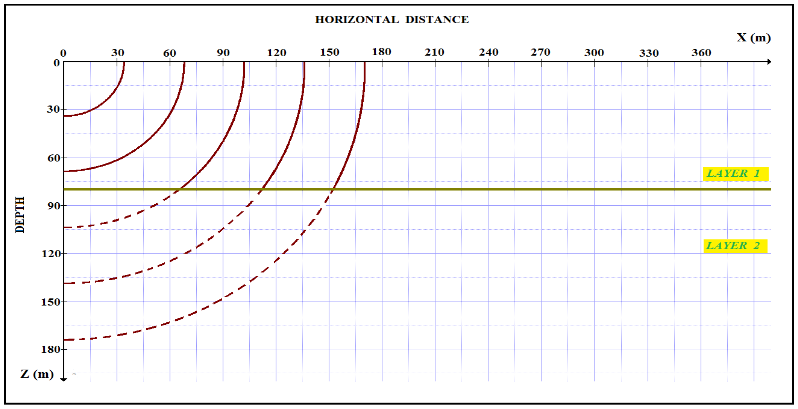

Fig. 9. Constructed wavefront model

8. CONCLUSION

These models show that the geometry of the raypaths and wavefronts of seismic waves propagating in a subsurface medium with velocity increasing linearly with depth can be constructed with circles. Raypaths deviate from the straight line paths associated with isotropic media. Wavefronts deviate from the shot-centred concentric circles and show greater travel distance in the vertical direction than in the horizontal. These variations will have effects (travel time, path length) in processing.

The models depict the actual geometries of the raypaths and wavefronts of seismic wave propagation in the region. They provide the theoretical basis for better approximation of the velocity structure and ray tracing of the Niger Delta lithofacies using relatively small layer thickness to accurately estimate geologically important seismic events such as changes in facies, fractures, faults, and unconformities and identify structural closures for better hydrocarbon target.

9. RECOMMENDATION

Recommendation is hereby made that Software Developers and Seismic Processors take into account the ray bending effect in the lithofacies in developing Niger Delta-specific software for processing and interpretation of seismic data in this region. Further studies on the lateral velocity variation should be carried out for the Niger Delta region in order to better understand the lateral and vertical anisotropy of raypaths and wavefronts for a 2-dimensional case in this region.

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COMPETING INTERESTS

Author has declared that no competing interests exist.

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