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Generalized Uncertainty Relations in Quantum Mechanics and the Principles of Completeness in Physics

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Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

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ABSTRACT

Focusing on the initial development of quantum mechanics, we will give a brief historical synopsis of the theory foundations, based on the Fourier framework and stating the philosophical conclusions inspired by that same mathematical formalism. We will then proceed, introducing an alternative way of describing the undulatory aspects of quantum entities, using local Gaussian Morlet wavelets. As we shall see, this change implies different philosophical interpretations about quantum reality and, even more, about the contemporary accepted differences between the quantum and the macroscopic realms. From these we will witness the formal and heuristic power of wavelet local analysis applied to the physical description of Nature. The ideas presented in this paper are initial standpoints of what can hopefully be expected to be a more mature and unifying physical theory, still undergoing development.

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1. INTRODUCTION

It is always a motive of wonder and surprise when a mathematical formulation adequately describes physical reality and inspires human thought about Nature's inner processes. And perhaps, even more important, when it deeply influences the relationship between man as a sentient creature and the world he tries to understand.

In this paper, we shall give an historical synopsis of the development of quantum mechanics based on Fourier's ontology [1], and focusing on the philosophical conclusions one usually finds whenever the theory is described or taught by the orthodox stand. Afterward, we will present an alternative way of describing the undulatory aspects of quantum entities using Gaussian Morlet wavelets. As we shall see, this will imply different philosophical conclusions about the quantum reality and even about the contemporary different accepted outlooks between atomic and macroscopic phenomena. The present work proceeds from past investigations [2] and pretends to clarify and develop the conclusions then drawn, hoping to stimulate further experimental and theoretical research on the foundations of quantum mechanics. It should be mentioned that some theoretical, mainly formal issues are still to be clarified, concerning the adequacy of the present theory to quantum phenomena. These do not seem to be fundamental drawbacks but mainly technical difficulties, only requiring more sophisticated use of wavelet analysis or further research on the applicability conditions of our proposals. We will say more about this in the Conclusion.

2. THE FOURIER FORMALISM AT THE BEGINNING OF QUANTUM MECHANICS

In 1927, in a lecture held at the Volta Conference at Como [3], Italy, Niels Bohr argued that the phenomenon of physical quantization put in evidence universal limitations on our knowledge about Nature. This was the "epistemological lesson taken from quantum mechanics", as he so eloquently put it. His narrative began by stating that the quantized nature of atomic systems was a consequence of the quantum of action given by Planck's constant *h*, that has a very small value, but which is, nevertheless, non-null. Due to the fact that *h* is different from zero it became possible to quantify energy, as Planck concluded earlier in 1900; to consider light behaving like corpuscles, as Einstein suggested in 1905; and to universally associate each particle with its own matter wave, as de Broglie conjectured in his Ph.D. thesis in 1924.

By 1927, at the time Bohr was fully developing his own philosophical views about Nature, the Planck-Einstein relation between energy and frequency, $E = h\nu$, was well known and de Broglie had already introduced his famous relation $p = h/\lambda$, between linear momentum and matter wavelengths, for all corpuscles. It was then quite remarkable that Niels Bohr, from the initial work of Heisenberg could develop, during the summer of 1927, a mathematical framework describing in a relational way, both corpuscular and undulatory aspects of quantum entities. Indeed, based on Fourier nonlocal analysis applied both to space and time, Bohr derived in a very beautiful way the same uncertainty relations Heisenberg told him about some months earlier, in March of 1927.

Bohr's basic idea was to associate spatial k and temporal ω frequencies, present in the fundamental phenomenological formulas of quantum physics, *viz*. de Broglie's

$$p = \hbar k \tag{1}$$

and Planck's

$$E = \hbar\omega \tag{2}$$

with infinite nonlocal harmonic plane waves, spreading over the entirety of space and time:

$$\psi = Ae^{i(kx-\omega t)}. \tag{3}$$

From it, by simple use of Fourier analysis

$$f(x) = \int_{-\infty}^{\infty} g(k)e^{ikx}dk, \tag{4}$$

and assuming a gaussian form for the distribution of the spatial frequency *k*,

$$g(k) = Ae^{-\frac{k^2}{2\sigma_k^2}} \tag{5}$$

One gets, by substitution,

$$f(x) \propto Ae^{-\frac{x^2}{2/\sigma_k^2}} \tag{6}$$

or

$$f(x) \propto Ae^{-\frac{x^2}{2\sigma_x^2}} \quad (7)$$

implying that

$$\sigma_x\ \sigma_k = 1. \quad (8)$$

For the time parameter, and following the same procedure, Bohr also derived

$$\sigma_t\ \sigma_\omega = 1. \quad (9)$$

Taking in consideration de Broglie and Planck formulas, (1) and (2), we finally get by substitution and for the ideal case:

$$\Delta x \Delta p_x = \hbar \text{ and } \Delta E \Delta t = \hbar, \quad (10)$$

These, in the general situation, simply written

$$\Delta x \Delta p_x \geq \hbar \text{ and } \Delta E \Delta t \geq \hbar, \quad (10')$$

giving the usual mathematical form for the Heisenberg uncertainty relations.

These relations were derived assuming that the only waves that have a well determined spatial frequency (that is, $\Delta k = 0$) and a well determined temporal frequency (that is, $\Delta\omega = 0$) are harmonic plane waves, that is, pure sine and cosine waves, spreading over the entirety of the universe, along whole space and time. An idealization which in fact corresponds to the kernel of Fourier analysis.

One immediate consequence of these implicit assumptions, contained in the Heisenberg relations, is that if a quantum particle, a neutron for instance, has a pure single energy value (that is, $\Delta E = 0$), and a single momentum value (that is, $\Delta p_x = 0$), then it is omnipresent in space and time. On the contrary, if one wishes to know the particle position and time with absolute precision (that is, $\Delta x = 0$ and $\Delta t = 0$), then the particle might have any value from an infinite number of momenta and energy values (that is, $\Delta p_x = \infty$ and $\Delta E = \infty$). Therefore, in Fourier and indeed Bohr's mathematical and conceptual framework we are logically prohibited from simultaneously knowing the exact position and the exact momentum of the neutron.

Bohr, with considerable philosophical insight, concluded that quantum behavior was not fully addressable using a causal space-time framework, as usually applied to macroscopic systems. At the Como conference, in 1927, he presented to an audience of physicists his own original interpretation about quantum phenomena, formulating his famous Complementary Principle, mathematically expressed, in the terms of Fourier analysis, as the Heisenberg relations.

As it was then formulated, the Complementary Principle states that, at the quantum level, the possibility of a space-time coordinate description is complementary to the possibility of a causal description. This can be seen from the uncertainty relations if one takes linear momentum and energy as major concepts in the dynamical causal description of an atomic entity behavior. The causal description will, of course, be masked by any effort to coordinate such an entity in space and time. Thus, in Bohr's initial and most fundamental views on complementarity, space-time coordination stands as mutual exclusive to a dynamical causal modeling of any sort.

At the same time, Max Born proposed a probabilistic interpretation for Heisenberg and de Broglie quantum waves, already associated with corpuscles. It is not thus difficult to accept that with Born probabilistic interpretation, the epistemological standpoint of Bohr eventually took the theory to a very extreme idealistic form. In fact, into what is today commonly known as the Copenhagen interpretation. In a way, it is very much due to Bohr's latest thoughts on the subject, that we now have infinite elusive probability waves devoid of any physical content, representing the state of knowledge of conscious observers about the world. These waves, although being only virtual and "unreal" entities, are nevertheless capable of interacting with slits and other physical entities, readily disposable for all practical purposes by means of a "collapse", once an observation has taken place. In fact, the metaphysical nature of the so-called psi waves implies that atomic objects are only potential entities, not really existing before being observed; only coming into existence due to the mysterious consciousness powers of human observers.

Louis de Broglie [4], in the celebrated fifth Solvay Conference held in September 1927, presented even then an alternative view to Bohr's platonic beliefs about quantum waves. Based upon his own earlier work, de Broglie thought that the quantum waves associated with atomic phenomena would be real physical perturbations in a subquantum medium, guiding or piloting, in a nonlinear way, the trajectory of corpuscles. This represented a causal nonlinear theory, where the statistically empirical aspects were only a consequence of the experimenter´s lack of knowledge about the initial velocities of particles. Although de Broglie's theory was quite sophisticated, thus offering domain for further improvement, it had a major drawback. de Broglie waves, despite being real physical entities, were still infinite Fourier sine and cosine oscillations, spreading throughout the whole of space and time. Consequently, Heisenberg indetermination scheme would still hold, although de Broglie reasoned that such restriction would be caused only by the measuring devices, imprinting a perturbation on the measured systems. The overall positivist view of Bohr and the Copenhagen school would however prevail and see this as an highly suggestive evidence for the presence of human knowledge constraints. And, even more, as evidence for the incomplete nature of atomic objects, metaphysically held from existence in their potential state. As a result, de Broglie's real pilot wave theory was refused, to be only later and partially reinstated by David Bohm pilot-wave theory. This, while preserving the causal part, therefore dispensing the wave's "collapse" by a conscious observer, would still place quantum waves in a metaphysical configuration space, respecting altogether the Fourier ontology.

3. WAVELET FORMALISM APPLIED TO QUANTUM MECHANICS

At the beginning of this century and giving sequence to de Broglie's research program, a nonlinear proposal for understanding quantum phenomena was presented [2]. This theory, formally included traditional quantum mechanics as a particular case, describing the same phenomena as orthodoxy, replacing Schrödinger's wave equation with a nonlinear master wave equation. Since we are now dealing with a nonlinear equation, the sum of two of its solutions is not, in general, a solution of the equation. Relevant to the present discussion is the fact that within the same approach for understanding Nature, and following de Broglie original suggestion, a quantum object is perceived as a complex inter-relational structure. A system constituted both by a corpuscular part and an extended undulatory part. The corpuscular part of the quantum particle, the acron, contains practically all the energy in the system, while the extended finite wave, named theta wave, subquantum wave, de Broglie wave, pilot wave or vacuum wave, is practically devoid of energy. Still, due to a nonlinear process, the subtle wave guides the high energetic corpuscle into the regions where the intensity of the theta wave is higher.

In this non-linear approach to quantum phenomenology - and here is the crucial point - the extended undulatory part is mathematically represented, not by a nonlocal harmonic infinite wave, but by a finite gaussian modulated wave, called a Morlet wavelet [5], written:

$$\theta = Ae^{-\frac{(x-vt)^2}{2\sigma^2}+i(kx-\omega t)}. \tag{11}$$

In this expression, v stands for the average velocity of the finite wave, which in general equals the velocity of the particle. The letter k stands for the spatial angular frequency, ω is the angular frequency and σ is the wavelet's spatial extension parameter, giving a value of how much the wave spreads throughout space, before decaying.

We stress that in this model there will be no infinite waves, because real physical perturbations need to be finite. Furthermore, as shown in ref. [2], the Gaussian Morlet wavelet is a solution of the nonlinear Schrodinger equation, the so called master equation:

$$-\frac{\hbar^2}{2\mu}\nabla^2\theta + \frac{\hbar^2}{2\mu}\frac{\nabla^2(\theta\theta^*)^{1/2}}{(\theta\theta^*)^{1/2}}\theta + V\theta = i\hbar\theta_t \tag{12}$$

Solutions of this nonlinear equation for some simple quantum systems, such as the harmonic potential and the hydrogen atom, were obtained by Rica da Silva [6], along with an interpretation of the experimental situation.

Using a wavelet for quantum wave representation has two major consequences:

The first is that there wouldn't be an absolute need for wave packaging in order to form a system localized in space and time, while still presenting both corpuscular and undulatory properties. The two kinds of properties, local and undulatory, would be intrinsically given from first principles.

The second consequence is that these same two kinds of properties would migrate to larger systems; these ones composed using wavelet packaging, corresponding, in a most natural way, to the physical situation where a larger system has smaller component systems. The mathematical formulation of such a composition allowing for the derivation of a new set of uncertainty relations, containing Heisenberg's relations as a particular case.

We now proceed to compare the two kinds of uncertainty relations. Those, resultant from nonlocal Fourier ontology, championed by orthodox quantum mechanics and to which Bohr gave so much philosophical meaning, and the ones coming from local analysis using wavelets and resulting from the above nonlinear quantum physics description.

First of all, it is interesting to see that the mathematical formulation in (11) contains the nonlocal harmonic plane wave used in orthodox quantum mechanics, as a particular case. It is thus heuristically richer. In fact, as the size of the wavelet increases, when $\sigma \to \infty$, one obtains:

$$\theta = Ae^{-\frac{(x-vt)^2}{2\sigma_{0x}^2}+i(kx-\omega t)} \underset{\sigma\to\infty}{\longrightarrow} Ae^{i(kx-\omega t)} \quad (13)$$

Which is the usual mathematical expression of an harmonic plane wave.

Following a process in all similar to Bohr's, it is possible to derive a general set of uncertainty relations, going beyond the Fourier ontology, assuming furthermore that a finite wave may have a single pure frequency

$$\theta(x) = e^{-\frac{x^2}{2\sigma_{0x}^2}+ik_0x} \quad (14)$$

Since any regular function, may be written in terms of wavelets, we have

$$\theta(x) = \int_{-\infty}^{\infty} g(k)\, e^{-\frac{x^2}{2\sigma_{0x}^2}+ikx} dk. \quad (15)$$

Where σ_{0x} represents the spatial spreading of the mother wavelet used in the wavelet packaging.

And choosing, as before, a gaussian form for the height function, $g(k)$, after some easy calculation we arrive at the general uncertainty relations:

$$\Delta x^2 = \frac{\hbar^2}{\Delta p_x^2+\frac{\hbar^2}{\sigma_{0x}^2}} \quad \text{and} \quad \Delta t^2 = \frac{\hbar^2}{\Delta E^2+\frac{\hbar^2}{\sigma_{0t}^2}}. \quad (16)$$

The plot of the first relation can be seen in Fig. 1, for different values of σ_{0x}

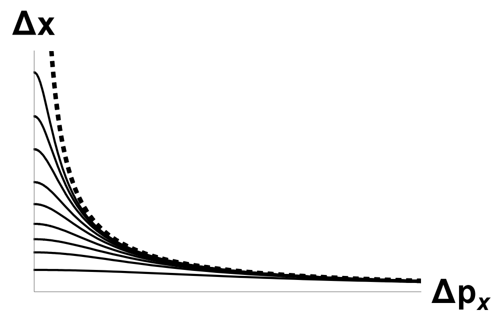

Fig. 1. First general uncertainty relation for different values of σ_{0x}

In Fig. 1 we show several instances for the first general uncertainty relation in (16). Each line reaches the vertical Δx axis on a different value of σ_{0x}, which increases upwards. The horizontal axis represents the linear momentum measure error Δp_x. The usual uncertainty relations, as is well known, would be represented by an hyperbola, the dashed line in the plot. It is clear that, for each curve, the largest value of Δx will be equal to σ_{0x} when $\Delta p_x = 0$, so that σ_{0x} also represents the maximum value for the position measure error Δx.

From these general uncertainty relations, one sees that when the size of the mother wavelet is relatively large $(\sigma_{0x} \to \infty)$, the general uncertainty relations transform into the traditional Heisenberg relations:

$\Delta x \Delta p_x \geq \hbar$ and $\Delta E \Delta t \geq \hbar$.

On the other hand, the general uncertainty relations (16) may also be written in the form:

$$\Delta p_x = \frac{\hbar}{\Delta x}\sqrt{1-\Delta x^2/\sigma_{0x}^2}, \quad (17)$$

$$\Delta E = \frac{\hbar}{\Delta t}\sqrt{1-\Delta t^2/\sigma_{0t}^2}\ , \quad (18)$$

in which we have considered $\sigma_{0x} = v\sigma_{0t}$ in the second relation.

From (17) it can be seen that if the space coordinate uncertainty Δx (or, inversely understood, the space coordinate accuracy) is of the same magnitude as the spatial spreading of the mother wavelet σ_{0x}, one can have a linear momentum accuracy as large as one wants, independently of the spatial coordinate accuracy. This happens even if the spatial required accuracy Δx is very high, in which case we must make σ_{0x} very small. That is, and following a similar reasoning for the time energy relation:

$$\text{If} \quad \Delta x \simeq \sigma_{0x} \quad \text{then} \quad \Delta p_x \simeq 0, \quad \forall\, \Delta x \quad (19)$$

And similarly,

$$\text{If} \quad \Delta t \simeq \sigma_{0t} \quad \text{then} \quad \Delta E \simeq 0, \quad \forall\, \Delta t. \quad (20)$$

This implies that, in (19) (and in (20)), for the particular limiting situation where we have $\Delta x \simeq \sigma_{0x} \simeq 0$ ($\Delta t \simeq \sigma_{0t} \simeq 0$) we may also have $\Delta p_x = 0$ ($\Delta E = 0$). In this natural way, one has regained the possibility of describing a physical situation, using spatial and temporal coordination and also dynamic information about the system. We have done a quantum measurement with an ideal zero error for the determination of both position and momentum and for the determination of both time and energy. In other words, we have accomplished $\Delta x = 0 \wedge \Delta p_x = 0$ ($\Delta t = 0 \wedge \Delta E = 0$), reaching beyond Niels Bohr Complementary Principle. In its mathematical formulation, this Principle claims that if $\Delta x = 0$ then $\Delta p_x = \infty$ (if $\Delta t = 0$ then $\Delta E = \infty$), and otherwise, if $\Delta p_x = 0$ then $\Delta x = \infty$ (if $\Delta E = 0$ then $\Delta t = \infty$). That is, we can either have $\Delta x = 0 \vee \Delta p_x = 0$ (or $\Delta t = 0 \vee \Delta E = 0$), but not all accuracies at the same time.

Significantly, if one takes in (17) the spatial accuracy Δx to be of an order of magnitude smaller than the mother wavelet spatial spreading σ_{0x} (Δt in (18) to be of an order of magnitude smaller than the mother wavelet temporal spreading σ_{0t}) , one gets the Heisenberg standard relations. That is,

$$\text{If } \Delta x \ll \sigma_{0x} \text{ and } \Delta x \simeq 0 \text{ then } \Delta p_x \to \infty\ , (21)$$

and similarly,

$$\text{If } \Delta t \ll \sigma_{0t} \text{ and } \Delta t \simeq 0 \text{ then } \Delta E \to \infty\ (22)$$

This means that in this situation there will be an inverse proportional relation between error and accuracy measurement for dynamical (undulatory) quantities and space-time coordination quantities.

Fig. 2 shows a 3D plot of Δx, accordingly to the first generalized relation in (16). The case shown in (19), that is, for the situation where we have low values for Δx and Δp_x (i.e. high accuracy for both physical measures) corresponds to the left low corner in the graphic, where we also have a low value for σ_{0x}. Note that it will always be $\Delta x \leq \sigma_{0x}$. The same would analogously result for the time energy relation.

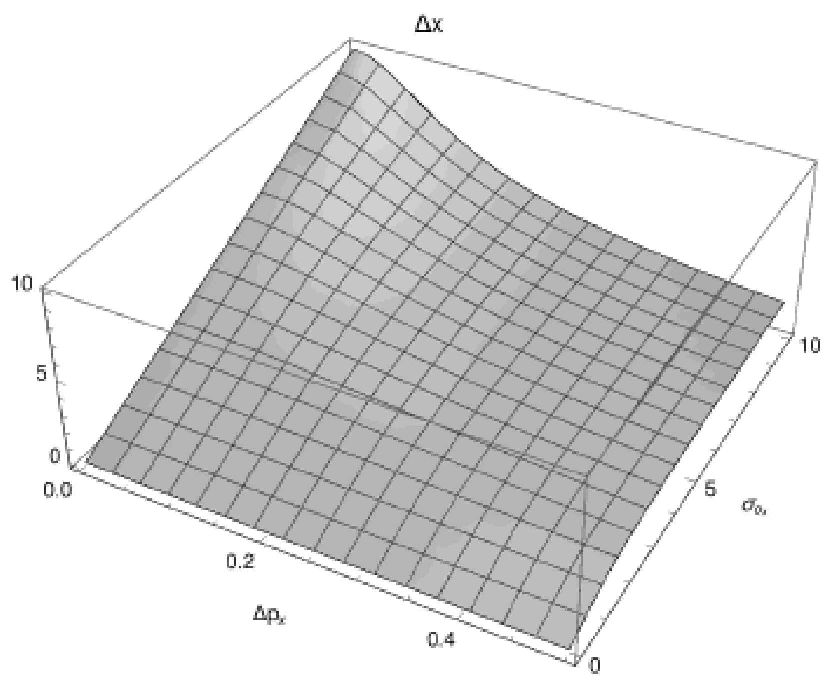

Fig. 2. Plot of Δx as a function of σ_{0x} and Δp_x

Inspired by what Niels Bohr thought a century ago while proposing his own interpretation of the Heisenberg's relations, we will now argue philosophically in a quite different way, providing our own interpretation based on the relations (17) and (18).

Since the Heisenberg relations may be seen as a particular case of the general uncertainty relations, presented in this paper, one may infer that if we change from nonlocal Fourier representation to local wavelet representation, it will stand to reason that the Complementary Principle will lose its universal status. This occurs even in the quantum case, since for the Complementary Principle to hold, it would be necessary to guarantee that the case considered in (19) and (20) never occurs at the atomic level, when in fact there are good reasons to suppose it does [7].

From all this, it thus seems reasonable to invoke a first alternative general principle stating the following:

> *Nature is an ontological complete structure, without any logical mutual exclusiveness, that is, an ontological unified structure which is consistent and intelligible*.

We again emphasize that the expressions (19) and (20) define a situation where causal dynamical description and space-time description are simultaneously feasible. Something very much akin to what seems to happen at the macroscopic scale.

Since for (17) and (18) we are still assuming the basic phenomenological fundamental formulas, (1) and (2):

$$p_x = \hbar k,$$

$$E = \hbar \omega,$$

and given that we have just sustained a sort of unification statement about Nature, we will now go a step further, hypothesizing the existence of only one physical description scheme, similarly applicable to both scales, atomic and macroscopic.

We consequently propose a second general principle stating that:

> *Nature can be physically described at all scales using both a causal undulatory scheme and a space-time local scheme, with variable accuracy dependence between the two descriptions*.

The accuracy dependence between those descriptions are of course given by (17) and (18), to which we may now call the Relations of Completeness, while naming the two posited statements the Completeness Principles about Nature. It should be noted that as long as Niels Bohr Complementary Principle is accepted only on an epistemological level, and therefore not as an absolute statement about Nature, it will be a correct particular case of the second Completeness Principle.

An obvious critique to the second statement is that things do not seem to adopt undulatory behaviors at the macroscopic scale. This however is not entirely true. Even if in most cases, macroscopically observable undulatory behaviors do not express themselves under our presently normal experimental use of objects, that does not mean that they aren't observable under other conditions. This is, for instance, the case of the so-called Doubochinski's pendulum and similar coupled oscillatory systems [8] showing quantified amplitude behaviours. Furthermore, Croca et al. [9] have found that the Titius Bode regularity in the solar system can be adequately explained using an undulatory formalism involving stationary pilot-waves.

4. CONCLUSION

We have seen how different ontological standpoints about the nature of quantum waves, and their respective mathematical descriptions, can lead to quite different philosophical conclusions about Nature. This seems to be indeed the case for wavelet local analysis once applied to quantum description, which has proven itself richer than the former Fourier scheme. Independently of how far long the reach of our proposed insights and interpretations might be, it seems undoubtable that accepting the realism and finiteness of quantum waves may lead to an unifying picture of Physics. Perhaps Louis de Broglie could have realized it, had he dismissed Fourier standard analysis to adopt wavelet analysis at first hand. It should again be noticed that the herein proposed pilot-wave theory using wavelet analysis, must be further worked for a set of cases already described by standard quantum mechanics. These include the relativistic treatment of the Master equation (12) and a more detailed description of the states of the hydrogen and helium atoms [6].

Another key point to be dealt with concerns the formal relation between the present theory and orthodox quantum mechanics. It should again be emphasized that although both theories uses undulatory formalisms, orthodox quantum mechanics describes Nature referring to Hilbert space, a pure conceptual configuration space. This, of course, comes from Bohr's choice of an idealistic ontology as to the nature of atomic entities. From a formal point of view, Hilbert space can be defined using Schrödinger's equation linear properties, that is, the fact that any linear composition of solutions is still a solution for that same equation. Furthermore, linearity of ordinary quantum mechanics enables the multiplication of a solution by a normalization factor, with $\int \psi^* \psi d^3x = 1$. This relation guarantees that the quantum particle exists somewhere in space. Although this linearity will not hold, in general, for the Master equation (12), it can be argued that Schrödinger's equation results in all situations where the non linear term in the Master equation is constant or null. This will depend on the steepness of the waveform intensity and, one should add, also on a very large sigma value, for which the wavelet spatial and temporal spreading becomes very large. Our own approach comes closer to prior ones, attempted by Vigier and de Broglie, in which the nonlinear term in their basic equation becomes of great importance in the very small region occupied by the particle [10]. A small distance away from the particle, in usual conditions such as in the atomic structure, the nonlinear term becomes negligible, and the linear description is recovered. One should finally add that the proposal for some nonlinear extension of orthodox quantum mechanics is not new. Several authors, such as de Broglie [10], Vigier [11], Smolin [12] and Weinberg [13] have proposed comparable modifications, such that strictly linear quantum mechanics, and its supporting Hilbert space, becomes recoverable in most situations. We hope to have contributed to a general physical theory, offering a wider and richer mathematical formalism applicable to all scales in Nature.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. Croca JR. Local analysis by wavelets versus nonlocal Fourier analysis. International Journal of Quantum Information. 2007;5(1&2):1-7. Croca JR, Garuccio A. Fourier ontology versus wavelet local analysis a test for the general validity of orthodox mechanics against the nonlinear causal quantum physics. Advanced Science Letters. 2010;3(2):1-4. Croca JR. Quantum indeterminism: A direct consequence of Fourier ontology, Proc. SPIE 8832, The Nature of Light: What are Photons? V, 88320Y; 2013. DOI: 10.1117/12.2025291 Available:http://dx.doi.org/10.1117/12.2025291
2. Croca JR. Towards a nonlinear quantum physics. World Scientific, London; 2003.
3. Bohr N. Como lectures, collected works, (North-Holland, Amsterdam, 1985). 1928;6.
4. de Broglie L. The current interpretation of wave mechanics: A critical study. Elsevier, Amsterdam; 1969.
5. Chui CK. An introduction to wavelets, academic press, New York; 1992. Kumar P. Wavelet analysis for geophysical applications. Rev. Geophysics. 1997;35: 385-412.
6. Rica da Silva A. Elementary nonlinear mechanics of localized fields and symmetry generated solutions of a nonlinear Schrodinger equation, both in A New Vision on Physics, J. R. Croca and J.E.F. Araújo, Eds., Center for Philosophy of Sciences, University of Lisbon; 2010.
7. Croca JR. Experimental violation of Heisenberg's uncertainty relations, talk at the 5th UK Conference on the Conceptual and Philosophical Problems in Quantum Physics, Oxford, September 1996; Croca, J.R., The Uncertainty Relations, Apeiron, 6, Nr.3-4, 151-165 (1999); Croca, J.R. and F. Selleri, Some Remarks Concerning Heisenberg's Indeterminacy Relations, Comm. Math. Theor. Physics. 1999;2:61-69.
8. Tennenbaum J. Amplitude quantization as an elementary property of macroscopic vibrating systems. 21st Century Science and Technology, Winter. 2005-2006;50.
9. Croca JR, Castro P, Gatta M, Cardoso A. Pilot-wave Gravity and the Titius Bode Law, New Horizons in Mathematical Physics, (Accepted for publication); 2017.
10. L. de Broglie. Une Tentative Causale et Non Linéaire de la Mécanique Ondulatoire, Gauthier-Villars, Paris; 1956.

11. Vigier JP. Explicit mathematical construction of relativistic nonlinear de Broglie waves described by three-dimensional (wave and electromagnetic) solitons "piloted" (controlled) by corresponding solutions of associated linear Klein-Gordon and Schrödinger equations. Found. Phys. 1991;21:125.
12. Smolin L. Quantum fluctuations and inertia. Phys. Lett. A. 1986;113:408.
13. Weinberg S. Precision tests of quantum mechanics. Phys. Rev. Lett. 1989;62:485.

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