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Mathematical Model to Quantify Air Quality: Indirect Measurement Approach

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Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

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ABSTRACT

The air is a basic need of all life on planet earth. Unfortunately, the quality of air has fallen down and the main impact of polluted air on human is related to severe health hazards. This is a burning problem to the developing economy of countries like Sri Lanka. Urbanization and rapid development in these countries are the main reasons for air pollution. Therefore, it is important to identify the levels of the air pollution in cities in order to predetermine actions to improve the quality of air.
There are lot of methods already available to measure the air quality, but these methods are based on direct measures of concentration of pollutants. The concentration cannot be measured continuously in countries like Sri Lanka by using direct measures due to lack of resources and financial support. Therefore, quantifying the levels of air pollution in cities using indirect technique is found practical.
The objective of this study is to develop a mathematical model to quantify air pollution using indirect measurements. Five most significant factors such as industries, population density, traffic intensity, green coverage and weather conditions are considered. The boundaries of the factors cannot be

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well defined. Therefore, fuzzy set theory is applied. Membership functions are defined for all the factors and those factors are combined using fuzzy operators. MATLAB program is used for the simulations.
In order to construct and to validate the mathematical model, twenty three major cities in the world are considered. Then, the ranks of cities are compared with the $\mathrm{PM}_{2.5}$ concentration levels. Finally, a case study is carried out by selecting Colombo Municipal Council region in Sri Lanka.

Keywords: Air quality; fuzzy set theory; fuzzy operators; rank.

1. INTRODUCTION

Clean air is an elementary prerequisite in all life on planet Earth. The composition of the present air varies considerably from the chemical composition of the original air, as it existed in pre-industrial era. The change in the global, chemical composition of the pre-industrial atmosphere due to human influence can be called as air pollution [1].

The main factors of air pollution are increase in population of metropolitan areas, high population density, growth of the industrial activities, oil refining, high rise buildings, increased vehicular movements and agricultural activities. These activities consume high amount of energy which release a large extent of pollutants such as carbon, nitrogen, sulfur compounds and particles into the air. The World Health Organization (WHO) revealed that 2 million [2] people die every year due to this salient hazard. It is one of the top ten killers in the world. In addition, air pollution causes many problems to human health and environment. Effects include increased several respiratory diseases like asthma, skin irritations, heart conditions and cancer. In addition global warming, acid rain, depletion of Ozone layer and effect on wildlife are observed.

Air pollution has been seriously emerged in Sri Lanka with no difference in other countries. Recent researches [3, 4, 5] reveal that Sri Lanka has perceived the significant growth in the level of ambient air pollution due to urbanization and increasing levels of industrialization. According to the available records [6] the level of air quality in Sri Lanka is moderate, but the medical records reveals that air pollution causes 7% increase in daily mortality, 30-35% in bronchitis and other respiratory diseases [7, 8]. Breathing contaminated air long term is aggravation of asthma, pneumonia, bronchitis, lung cancer and heart diseases which are long term effects. Lethargy, headaches, sore eyes and getting frequent flu are some of the short term symptoms of air pollution. The cost of medical care and treatments are very high to overcome these health hazards and it is a burden for the developing economy of Sri Lanka.

Compared to other countries in the world, Sri Lanka does not have heavy industries such as nuclear power plants and metal industry, but there are lots of small and medium scale industries in residential regions. Therefore, it is important to identify highly air polluted areas in Sri Lanka for urban planning and urban air quality management. When this is done authorities can recognize vulnerable cities to carry out anti air pollution projects such as planting trees in those areas in order to increase the green spaces.

World Health Organization (WHO) collects annual mean concentrations of particulate matter (PM_{10} and $\mathrm{PM}_{2.5}$) from 1600 cities covering 91 countries in the world [9] and identifies the air pollution level and health risk because particulate matter is an important indicator of long-term air quality and health risks. In addition, several researches [10, 11] have examined the emission of cities and rank the cities according to the air quality. However, the concentration of air pollutants cannot be assessed regularly in Sri Lanka due to non-existence of continuous operational monitoring stations. Sri Lanka is a country still developing slowly. The funds are not being allocated to increase facilities such as continuous pollutant measuring stations, to introduce other available equipments and to train technical staff. Therefore, the data cannot be measured precisely. Hence the public and relevant authorities do not have any method to recognize whether the quality of air we breathe is harmless. Most countries in the world have continuous monitoring stations and they give warnings to public over the media when the pollution levels go beyond the safe limits and recommend susceptible people such as people with heart disease and asthmatics to stay indoors. For example, Environmental Protection Agency (EPA) [12] in United States gathers concentrations of the major air pollutants such as carbon monoxide, ground level ozone, particulate matter and sulfur dioxide at more than a thousand places in United States and transformed those facts in to an air quality index (AQI) which can be easily understood by the general public. Therefore, developing a decision support index to quantify air pollution in Sri Lanka is an essential task. Due to lack of availability of reliable data and sources then the problem becomes complex. Some factors such as number of vehicles, number of factories and population density are proportional to the air pollution. These factors can be identified as the indirect measurements of air pollution.

The indirect measurements are uncertain. Therefore, it cannot define the exact boundaries of those measurements. Fuzzy set theory and fuzzy logic [13, 14, 15] are widely used areas in mathematical research which describe the problems which borders are not well-defined. This is widely used in nowadays to develop mathematical models of imprecise or vague information in the real world.

The main objective of this study is to suggest a new mathematical model to quantify air pollution indirectly. That kind of model can be used to rank the cities according to the air quality.

2. THEORETICAL BACKGROUND

Theoretical background of fuzzy theory and fuzzy operators, are discussed in this section.

Definition 0.1. A fuzzy set, A, in a universe of discourse, X, is a function of the form

$$f_A : X \to [0, 1]. \tag{0.1}$$

The function f_A is called the *membership function* of A, and for any x in X, $f_A(x)$ in $[0, 1]$ represents the grade of membership of x in A.

Definition 0.2. The *union*, $A \cup B$, of two fuzzy sets, A and B, is defined as the smallest fuzzy set containing both A and B. Its membership function is given by

$$f_{A\cup B}(x) = max\{f_A(x), f_B(x)\}, \qquad x \in X. \tag{0.2}$$

The *intersection*, $A \cap B$, is defined as the largest fuzzy set contained in both A and B. Its membership function is given by

$$f_{A\cap B}(x) = min\{f_A(x), f_B(x)\}, \qquad x \in X. \tag{0.3}$$

The *complement*, $-A$, of fuzzy set A is given by

$$f_{-A}(x) = 1 - f_A(x), \qquad x \in X. \tag{0.4}$$

Definition 0.3. The *algebraic product*, AB, is given by

$$f_{AB}(x) = f_A(x)\cdot f_B(x), \qquad x \in X. \tag{0.5}$$

Definition 0.4. Given fuzzy sets $A_1, A_2, ..., A_n$, the *convex combination*, B, is defined by

$$f_B(x) = w_1(x)f_{A_1}(x) + \ldots\cdots + w_n(x)f_{A_n}(x), \tag{0.6}$$

in which

$$\sum_{i=1}^{n} w_i(x) = 1,$$

and $0 \le w_i(x)$, for all $x \in X$. A special case of the above occurs when $w_i(x) = w_i$, a constant, $i = 1, \ldots, n$. In this case, B is called a *convex linear combination* of the A_i.

Definition 0.5. The *Hamacher operator*, H, which depends on p, is given by

$$H(A, B; p)(x) = \frac{f_A(x)\cdot f_B(x)}{p + (1-p)[f_A(x) + f_B(x) - f_A(x)f_B(x)]}, \qquad 0 \le p \le 1. \tag{0.7}$$

The interaction between A and B depends upon the parameter p; the degree of interaction decreases as p decreases. The Hamacher operator reduces to the algebraic product when $p = 1$.

Definition 0.6. The *concentration*, $CON(A)$, of a fuzzy set A is given by

$$f_{CON(A;a)}(x) = f_A^a(x), \qquad a > 1. \tag{0.8}$$

Concentration reduces the grade of membership of all elements x, with $f_A(x) < 1$. If reduced the grade of membership more then f_A is closer to 0.

Definition 0.7. The *dilation*, $DIL(A)$, of a fuzzy set A is given by

$$f_{DIL(A;a)}(x) = f_A^a(x), \qquad 0 < a < 1, \tag{0.9}$$

a is called the power of the operation. The inverse operation of concentration is called dilation.

3. METHODOLOGY

To model the air quality level in the cities using fuzzy sets, we select twenty three world cities including Colombo. Eighteen cities are selected to build the model and five are selected to validate the model. Finally, case study is carried out considering the main zones in Colombo Municipal Council region that is Colombo 1 - Colombo 15. Internet based survey is done considering the period between 2000 - 2014 in order to find the details of the world cities. The concentration values of pollutant $\mathrm{PM}_{2.5}$ are obtained from the World Health Organization database [9].

After surveying the literature and the sources which affect on the character of air is identified [16, 17]. Membership functions are defined for all the factors in order to apply the fuzzy set approach. The factors are industries, population density, traffic intensity, weather and green coverage. In this research the quality of air is modeled using only those factors. They are further separated in to sub-factors. Finally these factors are combined using fuzzy operators.

A new fuzzy operator is introduced to model the combine effect. This operator is defined by modifying the Hamacher operator and considering the concentration and dilation of fuzzy sets. The *modified Hamacher operator*, MH, defines the intersection of two fuzzy sets A and B by

$$MH(A,B;p,p_1,p_2)(x) = \frac{f_A^{\frac{1}{p_1}}(x) f_B^{\frac{1}{p_2}}(x)}{p + (1-p)[f_A^{\frac{1}{p_1}}(x) + f_B^{\frac{1}{p_2}}(x) - f_A^{\frac{1}{p_1}}(x) f_B^{\frac{1}{p_2}}(x)]}, \;\; 0 \le p \le 1, \;\; p_1, p_2 > 0. \tag{0.10}$$

For example, let us consider traffic intensity $C = 0.5$ and green area $E = 0.2$. Using these values algebraic product is $f_{CE}(x) = 0.1$, Hamacher operator is $H(C,E;0.5) = 0.1250$ and modified Hamacher operator is $MH(0.5, 0.2, 0.5, 0.8, 5) = 0.3311$. Therefore, algebraic product and Hamacher operator are not suitable to model the combine effect since it does not reflect the effect of green coverage. Hence these operators fails to model this type of problem. Selecting the modified Hamacher operator can highlight the effect of green area.

4. RESULTS AND DISCUSSION

4.1 Model Building

The membership functions are defined based on literature survey and experts ideas in the field. The selected factors and their membership values are described as follows:

[A.] Number of industries in the area: One indirect measurement of air quality is the number of industries exist in the area. A high density of industries in the area creates lot of problems and results for the poor air quality [18]. This factor is further subdivided into two categories, such as number of factories in the area and the number of power plants exist in the area. The fuzzy sets, i_1 the factory density in the area (factories/m^2) and i_2 the number of power plants in the area were defined by the membership function $A_1(i_1)$ and $A_2(i_2)$ respectively. The membership functions are defined as follow:

$$A_1(i_1) = \begin{cases} 1, & i_1 = 0 \\ 1 - 2\left(\dfrac{i_1}{150}\right)^2, & 0 < i_1 \leq 75 \\ 2\left(\dfrac{150 - i_1}{150}\right)^2, & 75 < i_1 \leq 150 \\ 0, & 150 < i_1. \end{cases}$$

$$A_2(i_2) = \begin{cases} 1, & i_2 = 0 \\ 1 - 2\left(\dfrac{i_2}{15}\right)^2, & 0 < i_2 \leq 7.5 \\ 2\left(\dfrac{15 - i_2}{15}\right)^2, & 7.5 < i_2 \leq 15 \\ 0, & 15 < i_2. \end{cases}$$

Here assume that the pollutant emission of all the factories are same. According to the expert ideas if at most 150 factories/m^2, then the risk of air pollution is high. The graphical presentation of the above fuzzy sets are shown in Figs.1 and 2. These two functions were combined as a linear combination and the weights are obtained from the experts knowledge [19].

$$A(i_1, i_2) = (0.6019 \times i_1) + (0.3981 \times i_2).$$

[B.] Population density: There is a direct relationship between air pollution and population density. The connection among pollution and population can differ from area to area. Let p equal the population density (persons/km^2) of the city and the membership function is defined as follows:

$$B(p) = \begin{cases} 1, & p \leq 2300 \\ 1 - 2\left(\dfrac{p - 2300}{69700}\right)^2, & 2300 < p \leq 37150 \\ 2\left(\dfrac{72000 - p}{69700}\right)^2, & 37150 < p \leq 72000 \\ 0, & 72000 < p. \end{cases}$$

Here assume that the population density up to 2300 persons/km^2 can be bearable and it is preferred for the good quality of air. The graphical presentation of the membership function is shown in Fig. 3.

[C.] Traffic intensity: High number of vehicles in the city may indirectly mean that has poor air quality. Let v equal the number of vehicles in the city limit (vehicles/km^2). The membership function for the traffic intensity is defined as follows:

$$C(v) = \begin{cases} 1, & v \leq 125 \\ 1 - 2\left(\dfrac{v - 125}{62775}\right)^2, & 125 < v \leq 31512.5 \\ 2\left(\dfrac{62900 - v}{62775}\right)^2, & 31512.5 < v \leq 62900 \\ 0, & 62900 < v. \end{cases}$$

Here assume that the pollution emission of all type of vehicles are same and vehicle density up to 125 vehicles/km^2 are preferred for the good air quality. The graphical presentation of the membership function is shown in Fig. 4.

[D.] Weather condition: Weather has positive and negative impact on the air pollution. This factor is further subdivided in to two categories. They are wind speed and temperature. High temperature will increase the effect of air pollution and the high wind speed is reduce the air pollution and increase the quality of air since wind carry the pollutant air away from it generated. Let w_1 denote the temperature ($^\circ\mathrm{C}$) and w_2 denote the wind speed ($\mathrm{km/h}$) in the city area were defined by the membership function $D_1(w_1)$ and $D_2(w_2)$ respectively. The membership functions are defined as below and the graphical presentations are shown in Figs. 5 and 6.

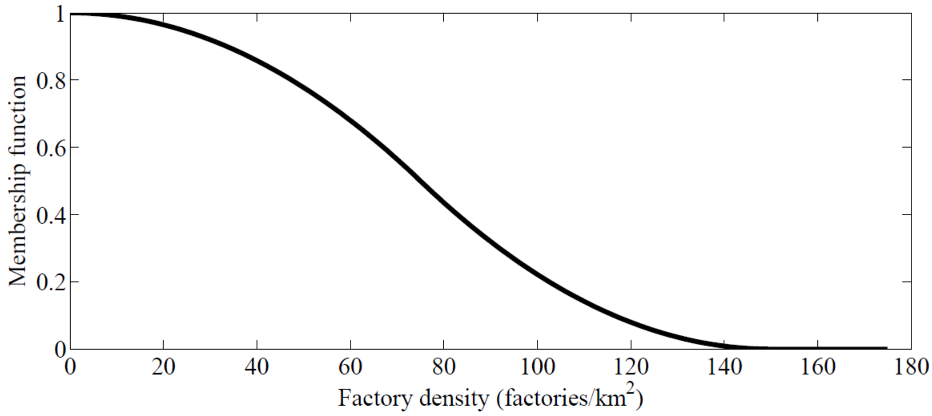

Fig. 1. Membership function for factory density

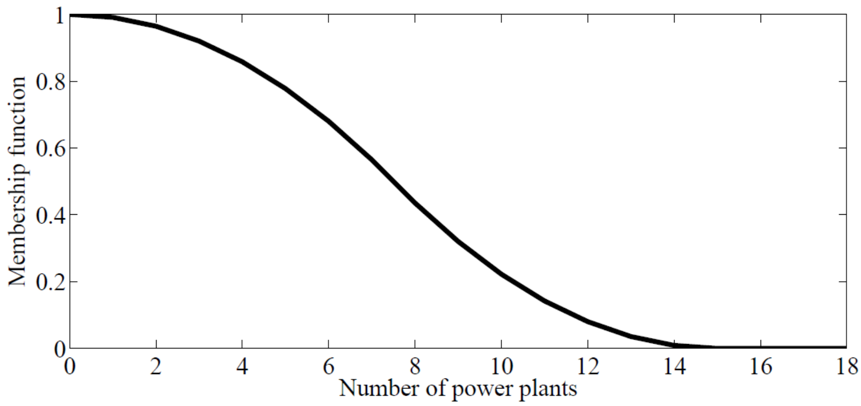

Fig. 2. Membership function for number of power plants

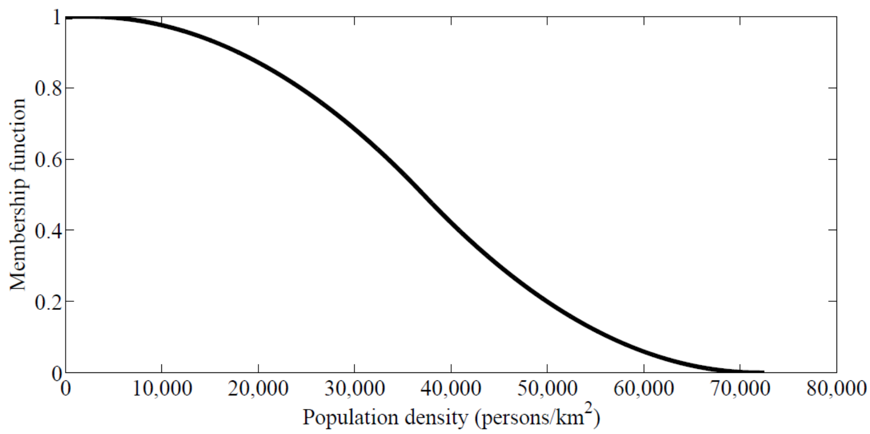

Fig. 3. Membership function for population density

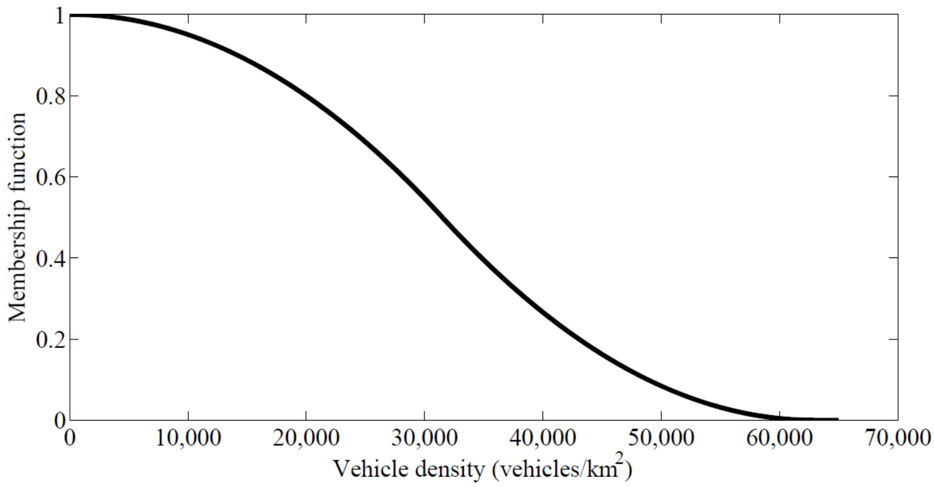

Fig. 4. Membership function for vehicle density

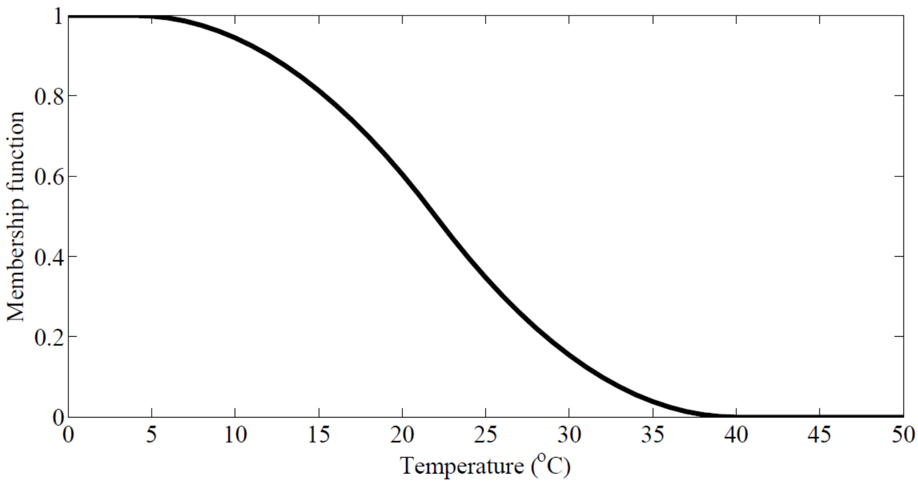

Fig. 5. Membership function for temperature

$$D_1(w_1) = \begin{cases} 1, & w_1 \leq 4 \\ 1 - 2\left(\dfrac{w_1 - 4}{36}\right)^2, & 4 < w_1 \leq 22 \\ 2\left(\dfrac{40 - w_1}{36}\right)^2, & 22 < w_1 \leq 40 \\ 0, & 40 < w_1. \end{cases}$$

$$D_2(w_2) = \begin{cases} 2\left(\dfrac{w_2}{40}\right)^2, & 0 \leq w_2 \leq 20 \\ 1 - 2\left(\dfrac{40 - w_2}{40}\right)^2, & 20 < w_2 \leq 40 \\ 1, & 40 < w_2. \end{cases}$$

Combined these two functions, which give equal weight to w_1 and w_2, yield

$$D(w_1, w_2) = D_1(w_1) \times D_2(w_2).$$

[E.] Green area: Trees absorb the substances in the polluted air and also it prevent the spread of the pollutants. Therefore, trees reduce the air pollution. Let g equal the green area per person (km^2/person) in the city limit. The membership function for the green coverage is defined as follows:

$$E(g) = \begin{cases} 0, & g \leq 9 \times 10^{-7} \\ 2\left(\dfrac{g - 9 \times 10^{-7}}{4.991 \times 10^{-4}}\right)^2, & 9 \times 10^{-7} < g \leq 2.5045 \times 10^{-4} \\ 1 - 2\left(\dfrac{0.0005 - g}{4.991 \times 10^{-4}}\right)^2, & 2.5045 \times 10^{-4} < g \leq 0.0005 \\ 1, & 0.0005 < g. \end{cases}$$

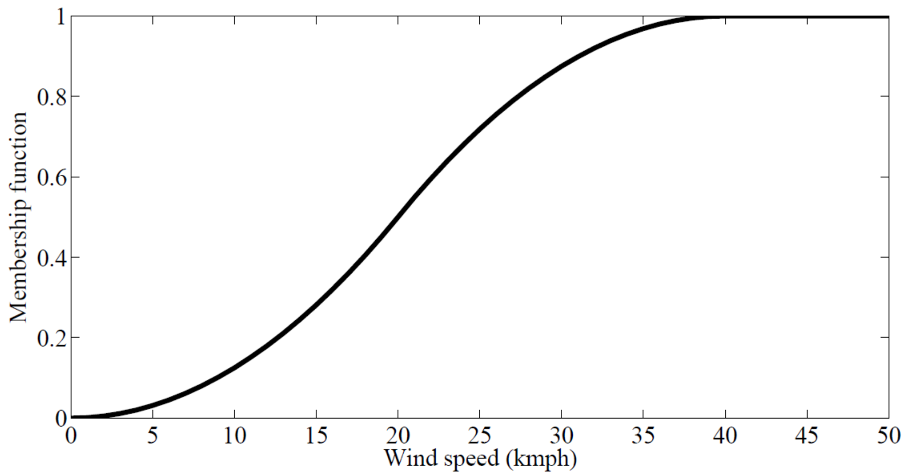

Fig. 6. Membership function for wind speed

Here assume that the optimum green coverage for a person is 0.0005. The graphical presentation of the membership function in Fig. 7.

Next combined the above membership functions using fuzzy operators. Selection of operators depend on the factors interactions. Several simulations are carried out considering various fuzzy operators and adjust the parameter values until the city reach to its EPA standard category. Possible fuzzy set function is obtained and it is as follows:

$$MH((MH(A^3 \times C^2, B, 0.5, 0.1, 0.2) \times D^{0.2})^{0.4}, \\ E, 0.5, 0.3, 5)^{0.08}.$$

First the interaction between A and C are identified and intersect them through the algebraic product. The cube of factor A is considered to give a more negative effect than the square of factor B. According to the expert knowledge factor A is the dominant factor for the air pollution [19]. Then the result combined with B via Modified Hamacher operator. The parameter values of Modified Hamacher operator are selected in order to give a more negative effect for the air quality. The factor D helps to improve the air quality somewhat. Therefore, combine the result through algebraic product and 0.4 power was taken to increase the air quality. Factor E plays a dominant role to improve the air quality. Hence combine the results up to now using Modified Hamacher operator. Finally 0.08 power of membership value is considered in order to improve the air quality. According to the model, selected countries and their air quality categories are shown in the Table . Table shows the propose air quality index and the categories.

4.2 Model Validation

According to the availability of data another five cities are selected in order to validate the model. Selected cities and categories are shown in the Table .

4.3 Case Study of Ranking the Zones in Colombo Municipal Council (CMC) Area According to Air Quality

Colombo is the traditional capital city of Sri Lanka. Since Colombo is the leading commercial, industrial and administrative city, it is a prominent fatality of air pollution. It is a busy and vibrant place with a mixture of modern life and colonial buildings. In the city limit the residential population is about 555,031. In addition, nearly 400,000 people visit Colombo Municipal Council [20] area daily due to existence of main hospitals, leading schools, universities, shopping complexes, main bus stands, main railway stations, industrial zones, factories, harbor and the airport. Nearly 275,000 vehicles enter in to Colombo almost every day. Those vehicles make traffic jams and therefore the emission of gases also make a significant contribution to air pollution. The electricity thermal power plant named “Kelanitissa” is also situated in this city. It emits smoke to the environment continuously. The only oil refinery in the country is also situated in Colombo. There is a noticeable increase in demand for fuel manufacture in this refinery. Its smoke consists of harmful gases, which plays an important role in air pollution. Thus, having better idea about the level of the air pollution will surely motivates the decision makers such as epidemiologist, environmental authorities, urban planners and traffic controllers. Therefore, it is important to identify the highly air polluted areas in Colombo Municipal Council region.

This case study is based on the geographical area governed by the Colombo Municipal Council i.e. Colombo 1 - Colombo 15. The zones are Fort (C1), Slave Island (C2), Kollupitiya (C3), Bambalapitiya (C4), Havelock Town/Kirilapone (C5), Wellawatte/ Pamankada/Narahenpita (C6), Cinnamon Gardens (C7), Borella (C8), Dematagoda (C9), Maradana/ Panchikawatte (C10), Pettah (C11), Hultsdorf (C12), Kotahena/Bloemendhal (C13), Grandpass (C14) and Mutwal/Modera/Mattakkuliya/Madampitiy (C15). The data of this area is collected from Department of Census and Statistics, Department of Meteorology and Department of Motor Traffic in Sri Lanka. Distribution of air quality sources in CMC are shown in the Table .

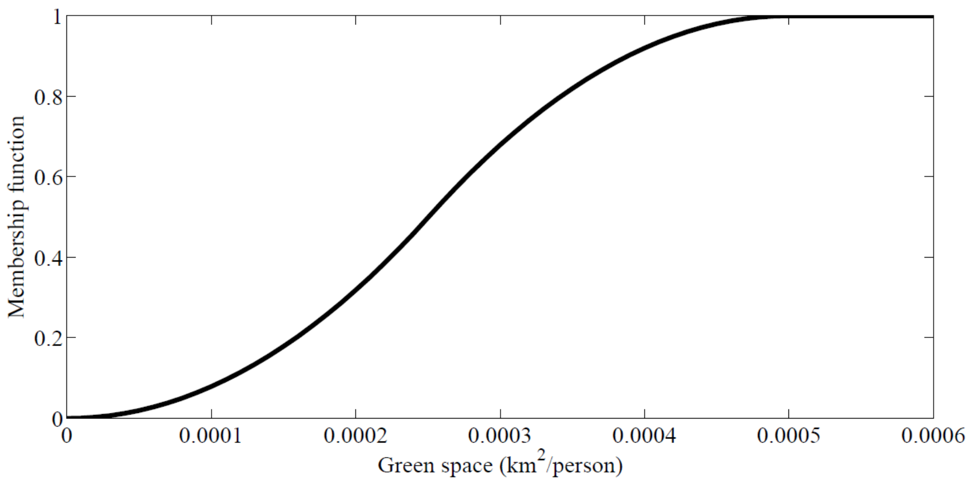

Fig. 7. Membership function for green area

Table 1: Air quality category of different countries

City	$\mathrm{PM}_{2.5}$ (μ/m^3)	EPA	Membership value	Category
Tokyo	14	2	0.7841	2
Mexico City	25	2	0.7953	2
New York	14	2	0.8281	2
São Paulo	19	2	0.7403	2
Mumbai	45	3	0.4207	3
Kolkata	43	3	0.5643	3
Shanghai	36	3	0.3390	3
Buenos Aires	16	2	0.6160	2
Delhi	153	5	0.0837	5
Los Angeles	20	2	0.8613	2
Osaka-Kobe	11	1	0.8896	1
Jakarta	21	2	0.6608	2
Beijing	56	4	0.1579	4
Cairo	73	4	0.2772	4
Dhaka	86	4	0.1504	4
Moscow	22	2	0.8376	2
Karachi	117	4	0.2692	4
Colombo	28	2	0.5783	2

Note: 1-Good; 2-Moderate; 3-Unhealthy for sensitivity group; 4-Unhealthy; 5-Very unhealthy; 6-Hazardous

Table 2: Air quality index

Category	EPA standard $(\mathrm{PM}_{2.5}(\mu/\mathrm{m}^3))$	Membership value
Good	00.0 - 12.0	0.8801 - 1.0000
Moderate	12.1 - 35.4	0.5701 - 0.8800
Unhealthy for Sensitive Group	35.5 - 55.4	0.3001 - 0.5700
Unhealthy	55.5 - 150.4	0.1503 - 0.3000
Very Unhealthy	150.5 - 250.4	0.0031 - 0.1502
Hazardous	250.5 - 500	0.0000 - 0.0030

Table 3: Model validation

City	$\mathrm{PM}_{2.5}$			Proposed model		
	μ/m^3	Rank	Category	Membership value	Rank	Category
Bangalore	45	1	4	0.2090	1	4
Singapore	17	5	2	0.5922	2	2
Bangkok	20	4	2	0.6022	3	2
Hong Kong	21	3	2	0.7050	4	2
Manila	22	2	2	0.7145	5	2

Note: 1-Good; 2-Moderate; 3-Unhealthy for sensitivity group; 4-Unhealthy; 5-Very unhealthy; 6-Hazardous

Table 4: Distribution of factors affecting the quality of air in CMC area

Zone	Power plants (%)	Factories (%)	Population (%)	Vehicles (%)	Green area
C1	0	20	2	20	9.68
C2	0	7	3	7	7.42
C3	0	8	4	8	5.18
C4	0	12	3	12	7.63
C5	0	2	9	2	2.42
C6	0	3	5	3	4.78
C7	0	1	2	1	9.23
C8	0	6	7	6	3.10
C9	100	11	16	11	1.39
C10	0	1	8	1	2.63
C11	0	12	1	12	36.08
C12	0	1	6	1	3.83
C13	0	4	19	4	1.11
C14	0	8	7	8	2.92
C15	0	4	8	4	2.57

Fuzzy theory can be easily used to rank the zones in CMC area than the traditional methods. The obtained results are shown in Table . According to the results the zones Fort, Bambalapitiya, Dematagoda, Pettah and Bloemendhal are the most vulnerable areas in terms of air quality. The zones Fort, Pettah and Bambalapitiya are identified as highly industrial areas with lot of road vehicles. In addition to the industries and road vehicles Dematagoda zone has a power plant. High population density in the area was the reason for Bloemendhal

Table 5: Ranks of zones in CMC area and their air quality categories

Zone	Membership value	Rank	Category
C1	0.0045	1	5
C2	0.5280	9	3
C3	0.3329	7	3
C4	0.0515	4	5
C5	0.6839	11	2
C6	0.8017	13	2
C7	0.8968	15	1
C8	0.5125	8	3
C9	0.0328	3	5
C10	0.7179	12	2
C11	0.0835	5	5
C12	0.8154	14	2
C13	0.0047	2	5
C14	0.3258	6	3
C15	0.6291	10	2

Note: 1-Good; 2-Moderate; 3-Unhealthy for sensitivity group; 4-Unhealthy; 5-Very unhealthy; 6-Hazardous

categorized as very unhealthy. Therefore new anti-air pollution projects such as limit the vehicles enter to these areas and add green belts could be introduced to these areas in order to improve the air quality. Cinnamon Gardens is the city which has cleaner air. Cinnamon Garden is a residential area with high percentage of green coverage and also the number of factories are low. New housing projects could be built in low air polluted areas like Cinnamon Gardens. These results are agreed with the prior knowledge of expertise.

5. CONCLUSIONS

In this study, fuzzy set approach is used to rank the cities according to air quality. Since the existing methods are costly and time consuming fuzzy set model can be used to make decisions using indirect measurements. Fuzzy operators are used to model the interaction between air quality factors. A new fuzzy operator is defined by combining the dilation, concentration of fuzzy sets and the Hamacher operator.

For the model construction and justification we select limited number of cities in the world due to limited accessibility of the data. Therefore, it is a limitation of this model. The model can be applied for other cities in Sri Lanka too in future.

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COMPETING INTERESTS

The authors declare that no competing interests exist.

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