Asian Research Journal of Mathematics

6(2): 1-9, 2017; Article no.ARJOM.29964
ISSN: 2456-477X

From Pascal Triangle to Golden Pyramid

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

Article Information

DOI: 10.9734/ARJOM/2017/29964
Editor(s):
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Complete Peer review History: http://www.sciencedomain.org/review-history/21026

Original Research Article

Received: 6th October 2016
Accepted: 29th May 2017
Published: 16th September 2017

Abstract

We introduce a "structure" of epic proportions – golden pyramid whose sacred geometry is "Fibonacci squared". In terms of mathematical beauty, the golden pyramid will perhaps be found to be comparable to Pascal triangle.

Keywords: Fibonacci sequences; golden pyramid; number genetics; Pascal triangle; pyramid symmetry; Teleois system; universal machine.

1 Introduction

The Pascal triangle, constructed by the French mathematician, physicist, and inventor Blaise Pascal (1623 – 1662), also called the Chinese triangle or Mount Meru, is not unknown to many mathematicians and scientists, see e.g. [1] to [5].We present the Pascal triangle in Table 1.1.

The Pascal triangle is known for its beautiful arithmetical properties, often studied alongside binomial expansions. It is also known that the Fibonacci sequence

$$F_n = 1,2,3,5,8,\ldots \quad (1.1)$$

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is obtainable from the Pascal triangle by taking diagonal sums. See [6] for yet another novel way of obtaining Fibonacci numbers from the Pascal triangle. In this paper we introduce a new numerical table, called golden pyramid, which has many mathematically interesting properties.

The creation of "Fibonacci sequences" is based on the argument that the sequence (1.1) is just a special arrangement of natural numbers thus any natural numbers can be similarly arranged, therefore there exist an infinite number of Fibonacci sequences, i.e. sequences satisfying the relations

$$\left.\begin{aligned} f_{n+1} &= round(\varphi f_n)\,; n \geq 1 \\ \varphi &= \frac{1+\sqrt{5}}{2} \end{aligned}\right\} \tag{1.2}$$

$$f_{n+2} = f_{n+1} + f_n, n \geq 1 \tag{1.3}$$

Table 1.1. Pascal/ Chinese triangle

										1										
									1		1									
								1		2		1								
							1		3		3		1							
						1		4		6		4		1						
					1		5		10		10		5		1					
				1		6		15		20		15		6		1				
			1		7		21		35		35		21		7		1			
		1		8		28		56		70		56		28		8		1		
	1		9		36		84		126		126		84		36		9		1	
1		10		45		120		210		252		210		120		45		10		1

When relation (1.2) is satisfied, relation (1.3) is satisfied by default. It becomes clear therefore that a Fibonacci sequence requires only one seed value called parent number. This seed value is recognized by a simple algorithm. Accept an integer z. Obtain $z\varphi$ and round off to integer y. Compute $y - z$, call it x. Obtain $x\varphi$ and round off to integer w. If $w = z$, then z is not a parent number. If $w \neq z$, then z is a parent number. We can also compute z/φ and round off to integer y, obtain $y\varphi$ and round off to integer w. If $w = z$, then z is not a parent number. If $w \neq z$, then z is a parent number. Parent numbers are therefore 1,4,7,9,12,14,17,20, etc.

2 Pyramid Construction

Given a Fibonacci sequence

$$F_n = f_1, f_2, f_3, f_4, f_5, \ldots \tag{2.1}$$

it is possible to determine the "genealogy" of this sequence. Suppose $f_1 = x + 1$ or $y - 1$. It is the business of number genetics, see [6], to determine which one of x or y is the mother cell of f_1. If x is detected as the mother cell of f_1, then f_1 is the daughter cell of x. Let $x = g_n$ in some Fibonacci sequence G_n. This means $(g_n,g_{n+1},g_{n+2},g_{n+3},g_{n+4},\ldots) + (1,2,3,5,8,\ldots) = F_n$. Similarly, if y is detected as the mother cell of f_1, letting $y = h_n$ in a Fibonacci sequence H_n, it follows $(h_n,h_{n+1},h_{n+2},h_{n+3},h_{n+4},\ldots) - (1,2,3,5,8,\ldots) = F_n$. Let's take the Fibonacci sequence

$$F_n = 7,11,18,29,47,\ldots \tag{2.2}$$

We compute $2f_1 - f_2 = 2(7) - 11 = 3 = h_{n-2}$. This means $h_n = 3\varphi^2 = 8$. Therefore, 7 = 8 – 1. This simply means (8,13,21,34,55,…) – (1,2,3,5,8,…) = 7,11,18,29,47,… It is important to note that $7 \neq 6 + 1$ because (6,10,16,26,42,…) + (1,2,3,5,8,…) = 7,12,19,31,50,… which is not the sequence (2.2). Number genetics provides the framework of logic within which the golden section is studied.

The rules governing pyramid construction are simple:

a) Sequences are written vertically, beginning with (1.1)
b) F_n with parent number $f_1 = c - 1$ is placed to the left hand side of c, with f_1 in the same level as c.
c) F_n with parent number $f_1 = c + 1$ is placed to the right hand side of c, with f_1 in the same level as c.

Following the above rules, Table 2.1 is produced.

Table 2.1. The golden pyramid

							1							
							2							
							3							
						4	5							
					7	6	8	9						
				12	11	10	13	15	14					
			20	19	18	16	21	24	23	17	22			
38	33	25	32	31	29	26	34	39	37	28	36	27	30	35

Due to space restrictions Table 2.1 gives the first eight levels of the pyramid, otherwise the pyramid is constructed to infinity.

3 Pyramid Analysis

Table 3.1 contains information on the eight pyramid levels presented in Table 2.1.

Table 3.1. Parameters for the first 8 pyramid levels

level	Number of elements (p)	Sum of elements (q)
1	1	1
2	1	2
3	1	3
4	2	9
5	4	30
6	6	75
7	9	180
8	15	480

Let P be the sequence of the number of elements in the pyramid levels. Beginning at third level, P is given by:

$$P = 1,2,4,6,9,15,\dots \tag{3.1}$$

Let Q be the sequence of the sum of elements in the pyramid levels. Beginning at third level, Q is given by:

$$Q = 3,9,30,75,180,480,\dots \tag{3.2}$$

We first analyse P. Note that p_1 and p_2 are f_1 and f_2 in (1.1); p_3 and p_4 are f_1 and f_2 in

$$4,6,10,16,26,\ldots \quad (3.3)$$

p_5 and p_6 are f_1 and f_2 in

$$9,15,24,39,63,\ldots \quad (3.4)$$

The sequences (1.1), (3.3), and (3.4) that supply f_1 and f_2 to P are obtained from the sequence (1.1). Let's have G_n = (1.1).

We see that P is given by

$$P = g_1^2, g_1g_2, g_2^2, g_2g_3, g_3^2, g_3g_4, \ldots \quad (3.5)$$

i.e.

$$P = 1^2, 1(2), 2^2, 2(3), 3^2, 3(5), 5^2, 5(8), 8^2, 8(13), \ldots$$

We see that P is comprised of the first and second terms of sequences of the kind

$$a, b, a+b, a+2b, 2a+3b, 3a+5b, 5a+8b, \ldots \quad (3.6)$$

therefore we can represent P as

$$P = a_1, b_1, a_2, b_2, a_3, b_3, a_4, b_4, \ldots \quad (3.7)$$

An examination of P reveals that

$$b_n = a_n + b_{n-1};\ n \geq 2 \quad (3.8)$$

For example, we see in (3.1) that 2 + 4 = 6; 6 + 9 = 15; etc. That the number of elements in pyramid levels (P) is governed by the sequence (1.1) is noteworthy. We now analyse Q. We have

P = 1,2,4,6,9,15,…
Q = 3,9,30,75,180,480,…

This relationship holds:

$$q_n = \frac{p_n p_{n+3}}{2};\ n \geq 1 \quad (3.9)$$

Using equation (3.9), Q is assembled to infinity and also using equation (3.5), P is assembled to infinity. Here we extend both P and Q:

P = 1,2,4,6,9,15,25,40,64,104,169,273,441,714,…
Q = 3,9,30,75,180,480,1300,3380,8736,22932,60333,…

The pyramid in Table 2.1 contains every (natural) number ≥ 1. Let x be the smallest integer in any given level, and let y be the largest. This means that such particular level is composed of all integers in the range x to y. For example, we can see from Table 2.1 that the lowest integer in level 7 is 16 and the largest is 24, and all integers between 16 and 24 are in that level. There is a most interesting pattern. The sequences whose parent numbers are the squares of terms of the sequence (1.1) also supply their first four terms as the minimum or maximum values in the pyramid levels. For example, the sequence (1.1) itself supplies f_1 to f_4 as maxima from level 1 to level 4; the sequence (3.3) supplies f_1 to f_4 as minima from level 4 to level 7; the sequence (3.4) supplies f_1 to f_4 as maxima in level 5 to level 8; the sequence

$$25,40,65,105,170,\ldots \quad (3.10)$$

supplies f_1 to f_4 as minima in level 8 to level 11; etc. More clearly, let's take G_n = (1.1). For odd n, g_n^2 is a parent number of F_n such that f_1,f_2,f_3, and f_4 are maxima in consecutive pyramid levels while for even n, g_n^2 is a parent number of F_n such that f_1,f_2,f_3, and f_4 are minima in consecutive levels. Simply put, the pyramid is a "Fibonacci squared" structure. Let us create U = maximum – minimum value per level beginning at the first level. For levels 1 to 3 we have zero since maxima = minima; for level 4: min = 4, max = 5 thus $u_4 = 5 - 4 = 1$; for level 5: min = 6, max = 9; thus $u_5 = 9 - 6 = 3$. Doing so, we assemble

$$U = 0,0,0,1,3,5,8,14,24,39,63,103,\ldots \quad (3.11)$$

The analysis of U is very interesting. It is assembled in phases. We first take G_n = (1.1). We extend it backward by two steps and we have $G_n' = 0,1,1,2,3,5,8,13,\ldots$ We see that $f_1 = g_n'^2$ (odd n) such that f_3,f_4,f_5,f_6+1 are consecutive elements of U. For instance take $g_3'^2 = 1 = f_1$. This means $f_3,f_4,f_5,f_6+1 = 3,5,8,14$ are consecutive elements of U, and we find them from u_5 to u_8. We take $g_5'^2 = 3^2 = 9 = f_1$. This means $f_3,f_4,f_5,f_6+1 = 24,39,63,103 = u_9$ to u_{12}. The sequence U is very important for detecting which zero in the null sequence

$$0,0,0,0,0,0,0,0,\ldots \quad (3.12)$$

is the mother cell of the parent number 1. From the above procedure, it is clear u_1 to u_4 are f_3,f_4,f_5,f_6+1 where F_n is the null sequence. Here, the parent number 1 is the daughter cell of the sixth zero in the null sequence. It might sound strange how we are here attempting to distinguish between zeros but this is important in further studies of the golden section.

4 Other Pyramids

The pyramid presented in Table 2.1 may be called the master pyramid since it contains every (natural) number ≥ 1. It should be noted that from any Fibonacci sequence we can create a pyramid. The pyramid in Table 2.1 is that of the sequence (1.1). For every F_n the same construction rules apply. It should be noted that the number of elements in the pyramid levels does not change and is therefore the same for every pyramid. For calculating the sum of elements in the levels of a pyramid of a given F_n, we below develop a universal machine.

First consider the pyramid of the sequence (1.1). Beginning at the $(i-2)^{th}$ level, $i \geq 0$; the sum of elements in the levels is given by:

$$Q' = 0,0,0,1,2,3,9,30,75,180,480,\ldots \quad (4.1)$$

We also modify P such that it begins at level $i = 1$. We have

$$P' = 1,1,1,2,4,6,9,15,25,40,64,104,\ldots \quad (4.2)$$

For any given F_n, we assemble Q'' such that

$$q_n'' = f_n p_n'; \; n \geq 1 \quad (4.3)$$

Let R be the sum of elements in the pyramid levels of a given F_n. R is computed from:

$$R = Q'' - Q' \quad (4.4)$$

As said, equation (4.4) computes the sum of elements in the pyramid levels of any given Fibonacci sequence F_n. This also includes F_n = (1.1). So equation (4.4) is a universal machine. Note that this equation holds when F_n is such that $f_1 = g_n + 1$; $n \geq 4$ in another Fibonacci sequence G_n. When F_n is such that $f_1 = g_n - 1$; $n \geq 4$, the machine

$$R = Q'' + Q' \tag{4.5}$$

is used. The universal machine therefore switches between two states (4.4) and (4.5) as dictated by number genetics.

We give one worked example.

Worked Example 4.1

Problem

Use universal machine to compute the sum of elements in the first ten levels of the pyramid of the Fibonacci sequence $F_n = 7,11,18,29,47,\ldots$.

Solution

First accept $P' = (4.2)$ as an argument. Now assemble Q'' by computing

$q_1'' = f_1 p_1' = 7(1) = 7$; $q_2'' = f_2 p_2' = 11(1) = 11$;
$q_3'' = f_3 p_3' = 18(1) = 18$; $q_4'' = f_4 p_4' = 29(2) = 58$;
$q_5'' = f_5 p_5' = 47(4) = 188$; $q_6'' = f_6 p_6' = 76(6) = 456$;
$q_7'' = f_7 p_7' = 123(9) = 1107$; $q_8'' = f_8 p_8' = 199(15) = 2985$;
$q_9'' = f_9 p_9'' = 322(25) = 8050$; $q_{10}'' = f_{10} p_{10}' = 521(40) = 20840$;

We therefore have

$$Q'' = 7,11,18,58,188,456,1107,2985,8050,20840,\ldots$$

Now we have to decide whether to use machine (4.4) or (4.5). We do number genetics for F_n. We have $2f_1 - f_2 = 2(7) - 11 = 3 = g_{n-2}$; thus $g_n = 8$. It follows $7 = g_n - 1$; therefore universal machine (4.5) applies. It follows

$$R = Q'' + Q'$$

$$= (7,11,18,58,188,456,1107,2985,8050,20840,\ldots) + (0,0,0,1,2,3,9,30,75,180,\ldots)$$

$$= 7,11,18,59,190,459,1116,3015,8125,21020,\ldots$$

In Table 4.1 we construct the pyramid.

Table 4.1. Pyramid of $F_n = 7,11,18,29,47,\ldots$

							7							
							11							
							18							
							29	30						
						46	47	49	48					
					75	74	76	79	78	77				
			127	122	121	120	123	128	126	125	124			
206	203	198	205	197	196	194	199	207	204	202	201	195	200	208

Table 4.1 can be used to confirm the above calculations.

It must be noted that in any level, maximum value – minimum value is the same for every pyramid, i.e. maxima – minima is given by the sequence (3.11). As pointed out on the pyramid in Table 2.1, there are special sequences that supply their first four terms as maxima or minima in consecutive pyramid levels. We here introduce Ladder theory for pyramid analysis.

5 Ladder Theory and Pyramid Analysis

Given any Fibonacci sequence F_n, we create the ladder:

$$\{f_1,f_2\} \rightarrow \{h_1,h_2\} \rightarrow \{j_1,j_2\} \rightarrow \{l_1,l_2\} \rightarrow \quad (5.1)$$

such that $h_2 - h_1 = f_2$; $j_2 - j_1 = h_2$; etc. Take F_n = (2.2) for instance. We have the ladder:

$$\{7,11\} \rightarrow \{17,28\} \rightarrow \{46,74\} \rightarrow \{119,193\} \rightarrow \quad (5.2)$$

Now given the pyramid of F_n = (2.2), we wish to predict the sequences that supply their first four terms as maxima or minima to the pyramid. We first take the ladder (5.2). Since in the sequence (2.2) it can be shown that 7 = 8 – 1, this implies that letting the sequence (2.2) be rung i = 0 in the ladder (5.2), the rungs 0,2,4,6,8, etc. provide their first four terms as minima to the pyramid levels. Now we create another ladder such that rung i = 0 is the sequence whose k-value ($k = f_2 - f_1$) is the parent number of rung i = 0 in the ladder (5.2). We thus create the ladder:

$$\{12,19\} \rightarrow \{30,49\} \rightarrow \{80,129\} \rightarrow \{208,337\} \rightarrow \quad (5.3)$$

in which the rungs 1,3,5,7, etc. provide their first four terms as maxima to the pyramid levels. The above procedure is followed for any pyramid of a given F_n. Number genetics plays a central role in this physics. More generally, let F_n, F_n', and G_n be Fibonacci sequences such that $f_2' - f_1' = f_1$. If $f_1 = g_n - 1$, $n \geq 4$, then in the ladder with F_n as rung i = 0, the sequences forming rungs i = 0,2,4,6, etc. provide their first four terms as minima to the levels of the pyramid of F_n, while maxima are provided by the first four terms of the sequences at rungs i = 1,3,5,7, etc. in the ladder with F_n' as rung i = 0. If $f_1 = g_n + 1$, $n \geq 4$, then in the ladder with F_n as rung i = 0, the sequences forming rungs i = 0,2,4,6,8, etc. provide their first four terms as maxima to the pyramid of F_n, while minima are provided by the first four terms of the sequences at rungs i = 1,3,5,7, etc. in the ladder with F_n' as rung i = 0.

Given any Fibonacci sequence F_n, we are now able to produce the ladders that control the geometry of the pyramid of F_n. It now suffices to present a higher concept. Let F_n and F_n' be as defined above, i.e. $f_2' - f_1' = f_1$. Let the ladder of F_n be ladder A and that of F_n' be ladder B. Assemble a sequence A' composed of the first four terms of the sequences forming the rungs i = 2,4,6,8, etc. in ladder A and a sequence B' composed of the first four terms of the sequences forming the rungs i = 1,3,5,7, etc. in ladder B. Compute $A' - B' = C'$. Let the segment $f_3,f_4,f_5,f_6,f_7,\ldots$ from F_n be represented by D'. When F_n is such that $f_1 = g_n - 1$; $n \geq 4$ in another Fibonacci sequence G_n ; then

$$D' - C' = E' \quad (5.4)$$

where

$$E' = 2,4,6,10,18,30,48,78,128,\ldots \quad (5.5)$$

When F_n is such that $f_1 = g_n + 1$; $n \geq 4$ in another Fibonacci sequence G_n ; then

$$C' - D' = E' \quad (5.6)$$

E' is very important as it reveals important geometry. If we write it as

$$E' = 2(1,2,3,5,9,15,24,39,64,\dots) \tag{5.7}$$

we see the geometry more clearly. The first four terms of E' are double the first four terms of the sequence 1,2,3,5,8,…; the next batch of four terms in E' are double the first four terms of the sequence 9,15,24,39,63,…; the next batch of four terms are double the first four terms of the sequence 64,104,168,272,…; etc. We call attention to the ladder:

$$\{1,2\} \rightarrow \{4,6\} \rightarrow \{9,15\} \rightarrow \{25,40\} \rightarrow \tag{5.8}$$

We see that E' is composed of double the first four terms of the sequences forming the rungs i = 0,2,4,6, etc. in the ladder (5.8). We therefore have a beautiful physics here. E' simply indicates that the sequence (1.1) is the principle of the golden section. Also note that P = (3.1), the sequence of the number of elements per level for any pyramid, is composed of the first two terms of the sequences in the ladder (5.8). In other words, a screenshot of the ladder (5.8) in current format simply gives P, i.e. P = 1,2,4,6,9,15,25,40,… The reader may therefore find ladder theory an interesting concept.

6 Pyramid Symmetry

Let's examine the sequence (4.2) which gives the number of elements per level beginning at the first level for any pyramid. We see that odd and even numbers are grouped separately, systematically in threes. A transition from odd to even or vice-versa occurs at Teleois positions, i.e. the levels 1,4,7,10,13,16, etc. This is the simple meaning of the Teleois: grouping into threes. It is noticeable that the pyramid has no permanent line of symmetry, i.e. the line of symmetry shifts after every three levels, i.e. at Teleois positions, alternating between 0 and 0.5. This is caused by unsymmetrical production of sequences at Teleois positions. The Teleois number system is also met on energy levels in Fibonacci sequences, see [6].*"In the structure of the Pyramid of Gizeh, the Teleois is so dominant that we are forced to believe that it was intentionally used to symbolize and record knowledge of the past… But more amazing, these same records were built into the temples of Tiahuanaco 4497 years BEFORE the Pyramid."* – Landone [7]. Hardy et al. [8], cited by Sherbon [9] say *"Understand the proportions of the atom and its electromagnetic frequencies and you can understand why the proportions of the Teleois were used."* Landone further states, *"When one finds these Teleois proportions in the solar system, in the human skeleton, in every geometric design which has been considered beautiful in all ages, in intervals of the musical scales, in the structures of every building of great beauty, and in the designs found inside of snowflakes, it suggests that there is some basic proportional principle of creation."*

7 Conclusion

The golden pyramid is a very important structure not only because it puts to proof the mathematical and philosophical reasoning employed in the assemblage of Fibonacci sequences, but it opens up new avenues of research. The immediate application of the golden pyramid is in communication.

Competing Interests

Author has declared that no competing interests exist.

References

[1] Stakhov AP, Aranson S. The Golden Non-Euclidean Geometry. World Scientific; 2016.

[2] Dilcher K, Stolarsky KB. A Pascal-Type triangle characterizing twin primes. Available:http://www.geometer.org/mathcircles/pascal.pdf [Last accessed October 6th 2016]

[3] Edwards AWF. Pascal's arithmetical triangle: The Story of a Mathematical Idea. Charles Griffin; 1987.

[4] Call G, Velleman D. Pascal's matrices. American Mathematical Monthly. 1993;100.

[5] Falcon S, Plaza A. The k-Fibonacci sequence and the Pascal 2-triangle. Chaos, Solitons, and Fractals. 2007;33.

[6] Mamombe L. Proportiones Perfectus law and the physics of the golden section. Asian Research Journal of Mathematics. [In Press]

[7] Landone B. Prophecies of Melchi-Zedek in the great pyramid and the seven temples. New York: The Book of Gold; 1940.

[8] Hardy D, et al. Pyramid energy. Clayton G.A: Cadake Industries; 1987.

[9] Sherbon M. Wolfgang Pauli and the fine-structure constant. Journal of Science. 2012;2(3).

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