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Laser Cooling of Radium Atoms, Computational Investigation

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Authors' contributions

This work was carried out in collaboration between both authors. Authors NAA and INSE, they designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript, managed the analyses of the study and managed the literature searches. We read and approved the final manuscript.

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Short Research Article

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ABSTRACT

In this work Matlab simulation of the atomic Liouville equation for spontaneous emissions was developed for laser cooling of Radium atoms. The atomic decay time driven was determined according to the photon account. The theory includes the mechanical light effect for to atomic structure. The study followed the stimulating solution of Optical Bloch Equations (OBE's) for four levels system in an atom trap, where the center of mass motion is described quantum mechanically. The results showed that the laser cooling approach to Maxwell's law, the reduction of the velocity leads to new distribution of velocities and the atoms moving towards the light source will resonate the light field (crossing resonance).

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Keywords: Laser cooling; radium atom; Optical Bloch Equations; Maxwell's law.

1. INTRODUCTION

The laser cooling has a principle that based on a momentum transfer from the laser beam to the atoms. When the laser light has the right frequency to be absorbed by the atom, then photons will transfer their momentum

$$\vec{p} = \hbar\vec{k} \tag{1}$$

where the wave vector $\vec{k}$ = ω/c and ω is the photon frequency. Therefore the wave vector should be pointing in to the laser beam direction. This moment will be at the opposite direction of the atom, so the atom is kicked back a bit [1]. When the photon is absorbed by the atom it will ended up above the ground state. There is no stability to the equilibrium because of its two levels system, and then the atom will decay to the ground state again in a while [2]. In this decay, a photon is emitted randomly.

When the atom comes to the ground state, the laser beam can be absorbed and another cycle starts again. If this cycle is done repeatedly, there is no preferred direction for the atom emits the photons in the decay. This means that the average momentum changes due to spontaneous emission is equal to zero, but the absorbed photons all transfer their momentum in the same direction opposite to the velocity of the atoms [3]. After a number of cycles, the atoms will be stopped or, in other words, cooled.

In Radium, see Fig. 1; the other states are metastable states which mean that the atoms can stay there for about a nanosecond. Radium has a $^1S_0$ ground state and a $^1P_1$ excited state and the transition has a wavelength of 483 nm. This transition is a potential cooling transition. Excited radium atoms decay to the metastable states $^3D_1$, $^3D_2$. This means that there are many cooling cycles the atom ends up in one of the metastable states. For an atomic beam coming from an oven at 730 K with an average speed of 220.3 m/s this is not much cooling. To improve this number, the atoms need to be brought back into the cooling cycle. This can be done with additional lasers tuned at the transitions between the metastable states and the $^1P_1$ state. These lasers pump the atoms from the metastable states to the excited state where they have a bigger probability to decay to the ground state than to decay in a metastable state again. In this way, the cooling cycle can continue [4,5].

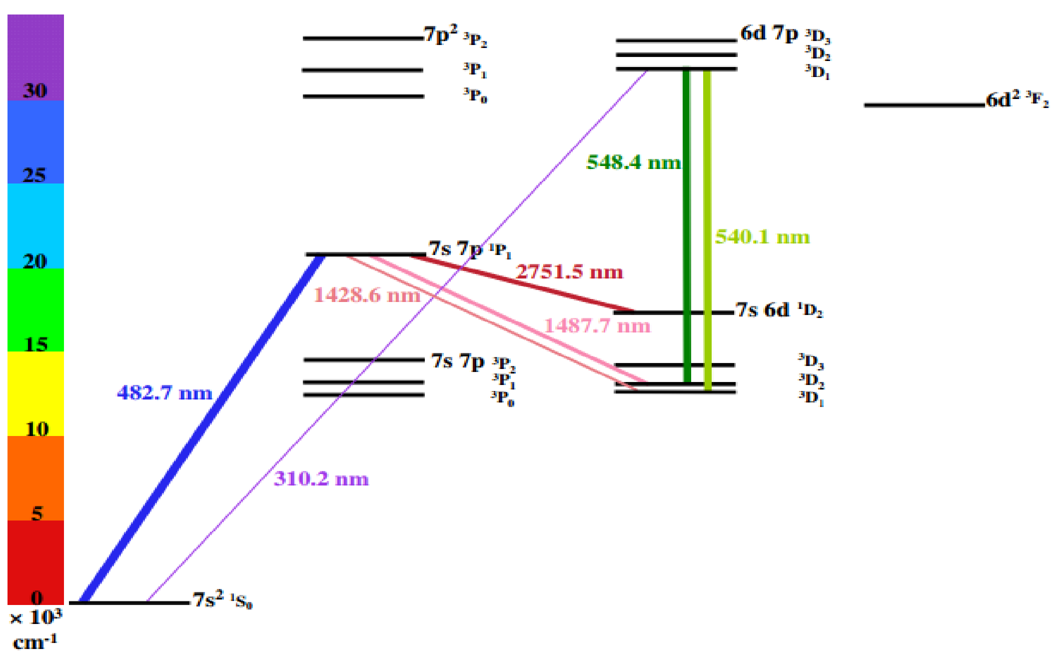

Fig. 1. The level scheme of ^{225}Ra atom

Table 1. The laser parameters needed for ^{225}Ra cooling

Transition	Wavelength [nm]	A_{2k} [10^6s−1]	Lifetime
$\lceil 2\rangle - \lceil 1\rangle$	483	166	6ns
$\lceil 2\rangle - \lceil 3\rangle$	1429	0.0014	0.7*e^{-3}s
$\lceil 2\rangle - \lceil 4\rangle$	1488	0 .25	3.95ms

2. THE MATHEMATICAL MODEL

The beam of Ra atoms from the oven has an initial Maxwell-Boltzmann distribution of speeds given by:

$$f(v) = \frac{v^3}{2\bar{v}^4}\exp\left(\frac{-v^2}{2\bar{v}^2}\right) \quad (2)$$

Where $\tilde{v}$, is average velocity
with $\tilde{v} = v_{\mathrm{rms}}$ depending on temperature.
where v_{rms} is the root mean square of velocity.

The velocities change during cooling and the Doppler shift is timed as well. We can use a 4-levels system for the calculations, including the hyperfine states, the atom-laser interaction is calculated by using the Optical Bloch Equations.

The force on an atom is calculated using the spontaneous scattering rate from the $^1P_1$ state. With this, we can calculate the change in velocity.

Since we start with a thermal velocity distribution of atoms from an effusive oven, we have to integrate over the full velocity range.

The Hamiltonian

The Hamiltonian of the atoms itself with eigenstates $|1\rangle, |2\rangle, |3|$ and $|4\rangle$ indicating the $^1S_0$,$^1P_1$,$^3D_1$ and $^3D_2$ states, respectively, is defined as: $\hat{H}_{atom}|a\rangle = \hbar\omega_a \; with \; a = 1,2,3,4$ [6].

The interaction Hamiltonian is:

$$\hat{H}_{int} = -\{\vec{D}_{12}.\vec{E}_{483} i[|2\rangle\langle 1|e^{-i\omega_{483}t} - |1\rangle\langle 2|e^{i\omega_{483}t} + \vec{D}_{23}.\vec{E}_{1429} i[|2\rangle\langle 3|e^{-i\omega_{1429}t} - |3\rangle\langle 2|e^{i\omega_{1429}t} + \vec{D}_{24}.\vec{E}_{1488} i[|2\rangle\langle 4|e^{-i\omega_{1488}t} - |4\rangle\langle 2|e^{i\omega_{1488}t} \quad (3)$$

where $\vec{D}_{12}$, $\vec{D}_{23}$ and $\vec{D}_{24}$ are the dipole matrix elements for the transitions between the upper state $|2\rangle$ and lower states $|1\rangle$, $|3\rangle$, $|4\rangle$, respectively.

Hamiltonian in matrix form is:

$$\hat{H} = \hbar \begin{pmatrix} \omega_{12} & \frac{\Omega_{12}}{2}e^{+i\omega_{483}t} & 0 & 0 \\ \frac{\Omega_{12}}{2}e^{-i\omega_{483}t} & 0 & \frac{\Omega_{23}}{2}e^{-i\omega_{1429}t} & \frac{\Omega_{12}}{2}e^{+i\omega_{483}t} \\ 0 & \frac{\Omega_{23}}{2}e^{+i\omega_{1429}t} & \omega_{23} & 0 \\ 0 & \frac{\Omega_{24}}{2}e^{+i\omega_{1488}t} & 0 & \omega_{24} \end{pmatrix} \quad (4)$$

where the zero energy is set to $|2\rangle$ and $\omega_{12} = \omega_1 - \omega_2$, $\omega_{23} = \omega_3 - \omega_2$ and $\omega_{24} = \omega_4 - \omega_2$. The Rabi oscillation frequencies are defined as:

$$\hbar\Omega_{12} = \vec{D}_{12} \,.\, \vec{E}_{483}$$
$$\hbar\Omega_{23} = \vec{D}_{23} \,.\, \vec{E}_{1429} \quad (5)$$
$$\hbar\Omega_{24} = \vec{D}_{24}.\vec{E}_{1488}$$

Density Matrix

$$\hat{\rho} = \sum_{\mathrm{a,b=1,2,3,4}} \rho_{ab} \; |\mathrm{a}\rangle\langle\mathrm{b}| \quad (6)$$

The trace Tr ($\hat{\rho}$) = 1 means that the probability of finding atom in any of the eigen states is 1. The off-diagonal elements describe the coherences between two states [7].

The time evolution of the density matrix is expressed in the Liouville equation:

$$\frac{d\hat{\rho}}{dt} = -\frac{di}{\hbar}\left[\hat{\mathrm{H}}, \hat{\rho}\right] + \hat{L}_{damp}(\hat{\rho}) \quad (7)$$

where $\hat{L}_{damp}$ is of the form:

$$\hat{L}_{damp}(\hat{\rho}) = -\frac{1}{2}\sum_m\left[\hat{C}_m^{\dagger}\hat{C}_m\hat{\rho} + \hat{\rho}\hat{C}_m^{\dagger}\hat{C}_m - 2\hat{C}_m\hat{\rho}\hat{C}_m^{\dagger}\right] \quad (8)$$

The first two terms describe the decay from the excited states and the third term describes the decay into the lower levels. The finite linewidth of the lasers can also be included. The terms $\hat{C}_{pq}$ are:

$$\begin{aligned}
\hat{C}_{21} &= \sqrt{A_{21}}|1\rangle\langle 2| \\
\hat{C}_{23} &= \sqrt{A_{23}}|3\rangle\langle 2| \\
\hat{C}_{24} &= \sqrt{A_{24}}|4\rangle\langle 2| \\
\hat{C}_{483} &= \sqrt{2\Gamma_{483}}|1\rangle\langle 1| \\
\hat{C}_{1429} &= \sqrt{2\Gamma_{1429}}|3\rangle\langle 3| \\
\hat{C}_{1488} &= \sqrt{2\Gamma_{1488}}|4\rangle\langle 4|
\end{aligned} \quad (9)$$

A_{2k} are the Einstein coefficients for the spontaneous decay. They are related to the lifetime of the excited state as $\tau^{-1} = \sum_k A_{2k}$ and can be found in Table (1). The Γ's are the linewidths of the lasers. Equation (7) can be written as:

$$\frac{d\hat{\rho}}{dt} = L\hat{\rho}(t) \quad (10)$$

with

$$L\hat{\rho} = -\frac{i}{\hbar}\left[\hat{\mathrm{H}}, \hat{\rho}\right] + \hat{\mathrm{L}}_{damp}(\hat{\rho}) \quad (11)$$

The total system can be transformed into the rotating frame of the laser light to get

$$\widehat{H}` = \hbar\begin{pmatrix} \Delta_{483} & \Omega_{12}/2 & 0 & 0 \\ \Omega_{12}/2 & 0 & \Omega_{23}/2 & \Omega_{24}/2 \\ 0 & \Omega_{23}/2 & \Delta_{1429} & 0 \\ 0 & \Omega_{24}/2 & 0 & \Delta_{1488} \end{pmatrix} \quad (12)$$

with $\Delta_{483} = \omega_{483} - \omega_{12}$, $\Delta_{1488} = \omega_{1488} - \omega_{23}$ and $\Delta_{1429} = \omega_{1429} - \omega_{24}$, specifying the detuning of the lasers with respect to the atomic transitions [8].

Optical Bloch Equations(OBE's)

The Liouville equation (10) can be seen as a system of linear transformations by writing:

$$\vec{\rho} = (\rho_{11}, \rho_{12}, \dots, \rho_{43}, \rho_{44})$$
$$\frac{d\vec{\rho}}{dt} = \sum_j M_{ij}\vec{\rho}_j \quad (13)$$

where $\rho_{ab} = \langle a|\hat{\rho}|b\rangle$. These equations are the Optical Bloch Equations, the matrix M is completely determined by L and it contains all the physical parameters needed to describe the interaction of the atoms with the laser light. Equation (13) can be written in linear form as:

$$\frac{d\rho_{rs}}{dt} = \sum_{kj} M_{rs,kj}\,\rho_{kj} \quad (14)$$

For an n-level system, the matrix M is of size n × n and is called the Liouville matrix. The matrix M is defined as:

$$M_{rs,kj} = -\frac{i}{\hbar}\left(\widetilde{H}_{rk}\,\delta_{js} - \widetilde{H}_{js}^{\dagger}\delta_{rk}\right) + \sum(C_m)_{rk}\,(C_m^{\dagger})_{js} \quad (15)$$

where the effective Hamiltonian $\widetilde{H}$ is introduced as

$$\widetilde{H} = \widehat{H}` - \frac{i}{2\hbar}\sum_m \hat{C}_m^{\dagger}\,\hat{C}_m \quad (16)$$

This set of complex linear differential equations can be solved numerically on a computer.

3. THE SIMULATION WORK

The rate of cooling can be easily calculated from the rate of scattered photons by spontaneous emission from the $^1P_1$ state [9,10]. This rate is:

$$N = \frac{\rho_{22}}{\tau} = \rho_{22}\sum_k A_{2k} \quad (17)$$

There are two main flow charts to describe the simulation steps; Fig. 2; describes the Liouville script and Fig.3; describes the cooling script.

4. RESULTS AND DISCUSSION

Fig. 4 shows the number of atoms in the ground state as a function of the final velocity for an apparatus length of 80 cm. This is the general picture, the thick line shows the initial Maxwell-Boltzmann distribution, and the thin line is the final distribution of the cooled atoms, the final velocity reached to 200 m/s. By changing Rabi frequency from 1 to 10 x10^6 rad/s, one observed a shift in the velocity approximately 50 m/s.

This figure tells that most of the cooling can only take place in part of the apparatus and that it may be more efficient to broaden the lasers than to add more length to the apparatus, by changing the detuning of the cooling laser basically selects another velocity class that will be cooled. Selecting a part of the Maxwell-Boltzmann distribution with many atoms also cools many other atoms, but maybe not to very low speeds. Selecting a part of the MB-distribution with low initial velocities and fewer atoms cools less atoms, but yields lower final speeds.

Fig. 5; indicates that the detuning change of the lasers must happen in a way that all the lasers are still on the same velocity class resonance. This showed that there is proportionality between the initial velocity and the final velocity for all steps. As example; when the cooling laser is detuned for a velocity of 100 m/s and the re-pump lasers for 500 m/s, there will be no re-pumping for the cooled atoms thought there is no much cooling will happen at all. The re-pump lasers may slightly be red detuned considering the cooling laser because they can start working after some cooling has been done.

Fig. 6; shows that for different Rabi frequencies of the cooling transitions. 483 nm laser is detuned for velocity of 200m/s and that atoms were cooled from the velocity class. As the cooling power increases, the peak shifts to lower velocities; this indicates that the re-pump intensities are not high enough (out of resonance).

Fig. 7; shows the relation between the velocity loss and time in microsecond, making kinetic energy small.

The amount of cooling per time step is plotted as a function of time. At t = 0, the atoms are at the oven. As time passes, they have moved towards the end of the apparatus. Also this figure shows the photon return as a function of time for a Radium atom initially tuned to the laser frequency (Δv = 0 MHz). The photon production rate increases at the beginning, due to optical pumping of the Radium population to the $^1P_1$ state and then decreases as radiation pressure pushes the atoms away from the laser frequency. This figure indicates that atoms moving towards the light source will resonate the light field, according to the equation:

$$\omega_{Doppler} = \omega_{laser} - k_{laser}\, v$$

This means that there is a change in Doppler shift.

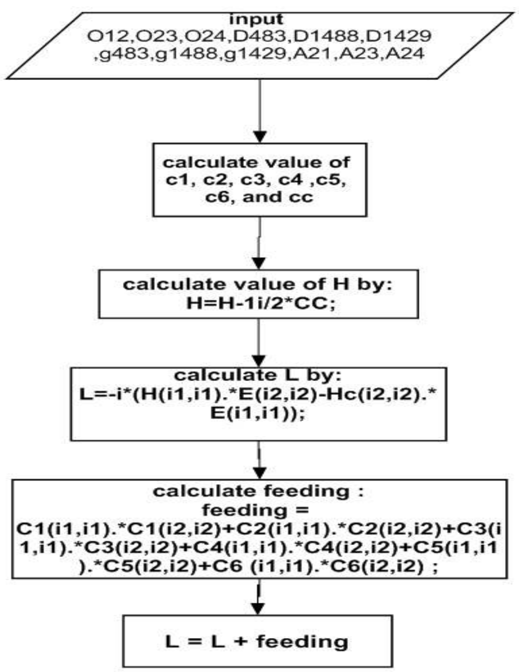

Fig. 2. Liouville script

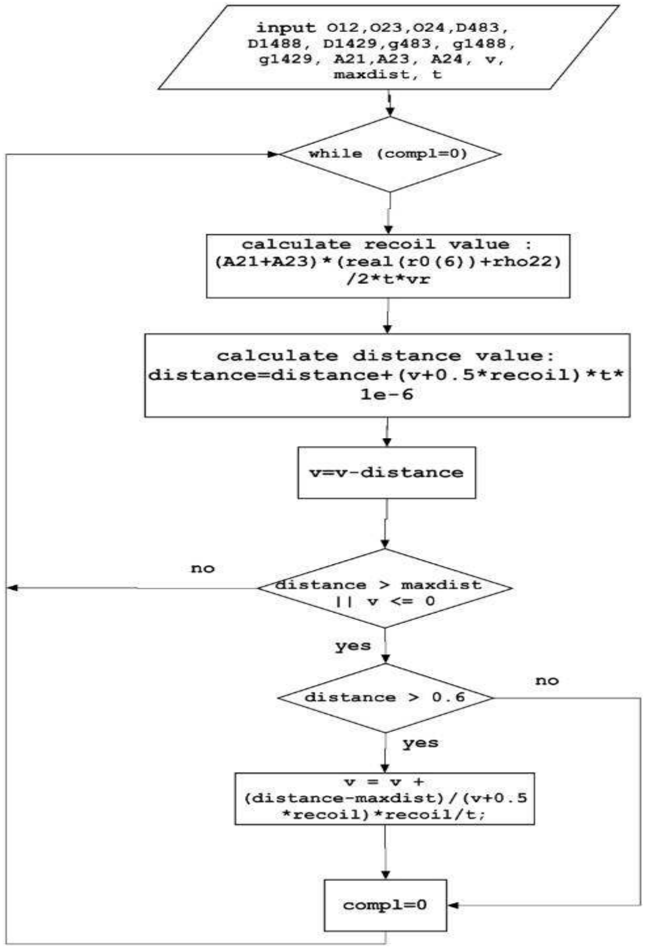

Fig. 3. Cooling script

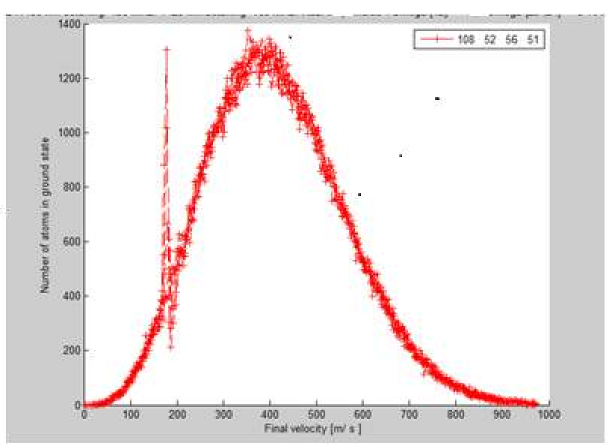

Fig. 4. The relation between numbers of atom and final velocity

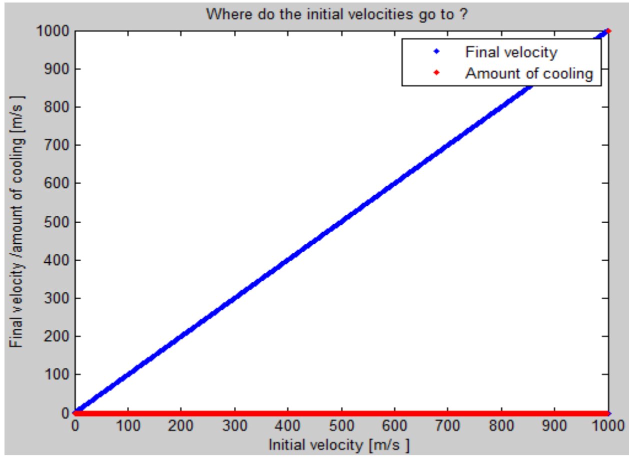

Fig. 5. The relation between the initial velocity and the final velocity

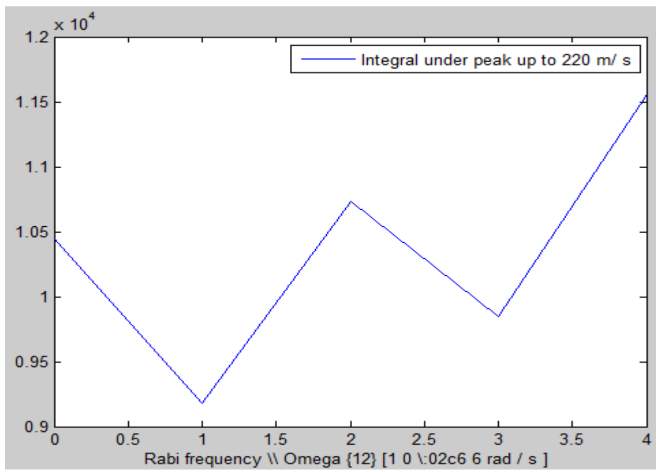

Fig. 6. The relation between the number of atoms and rabi frequency

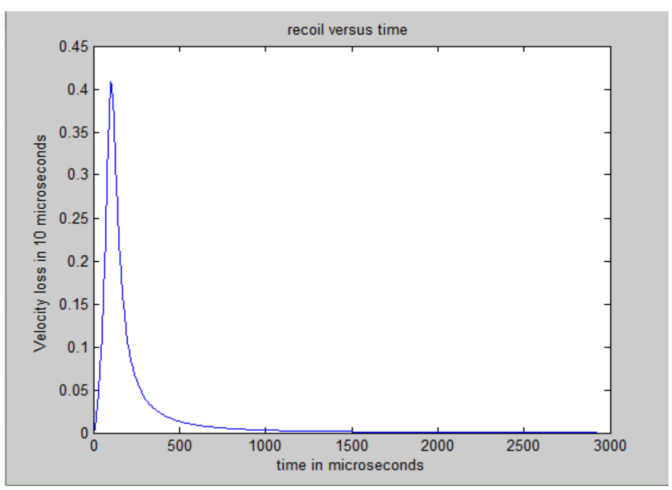

Fig. 7. The Relation between velocity loss and time in microsecond

5. CONCLUSIONS

The velocity distribution curve showed a narrow distribution near the lowest energy, due to the detuning of the laser, which means efficient cooling of the atoms.

In the future work, this open a route to study and manipulation of Radium atom in laser cooling regime.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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