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The Existence of Blow-up Solution to Nonlinear Schrödinger Equations with First-order Spatial Derivative Terms

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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Abstract

In this paper, we study the existence of blow-up solution to nonlinear Schrödinger equations with first-order spatial derivative terms at finite time.

Keywords: Nonlinear Schrödinger equation; first-order spatial derivative; blow-up solution.

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1 Introduction

In the present paper, we consider the nonlinear Schrödinger equation with first-order spatial derivative terms

$$\begin{array}{rcl} \phi_t & = & i\alpha\phi_{xx} + \beta\phi^2\bar{\phi_x} + \gamma|\phi|^2\phi_x + ig(|\phi|^2)\phi, \\ \phi(0,x) & = & \phi_0(x). \end{array} \tag{1.1}$$

As far as we know, there is little article to study the global solution and blow-up solution to (1.1) which includes first-order spatial derivative terms $\beta\phi^2\bar{\phi_x} + \gamma|\phi|^2\phi_x$, it is studied only when the lack of the first-order spatial derivative terms . For the case with first-order spatial derivative term, the existence of global solution is an open question even one dimensional space. We investigate the existence of blow up solution of (1.1).

Let me called some results on the case without first-order spatial derivative term. The nonlinear Schrödinger (NLS) equation provides a canonical description of envelope dynamics of a quasi-monochromatic plane wave propagation processes which are negligible. The dynamics are linear within short propagation distance and short time, but cumulative nonlinear interactions lead to an important modulation of the wave amplitude on large spatial and temporal scales.

In optics, it can also be considered as the extension to nonlinear media of the paraxial approximation, which used for linear waves propagating in some random medium. However, by I.P. Kaminow [1], we know that single-mode optical fibers are not really single-mode, but actually bimodal according to the presence of birefringence. This birefringence can influence the way in which an optical evolves during the propagation travel along the fiber. Indeed, it occurs that the linear birefringence makes a pulse split into two, and nonlinear birefringent traps them together against splitting. C.R. Menyuk [2,3] showed that the evolution of two orthogonal pulse envelopes in birefringent optical fibers, which can be governed by the following coupled nonlinear Schrödinger system:

$$i\phi_t + \phi_{xx} + (|\phi|^2 + e|\psi|^2)\phi = 0, \tag{1.2}$$

$$i\psi_t + \psi_{xx} + (e|\phi|^2 + |\psi|^2)\psi = 0, \tag{1.3}$$

where e is a positive constant and depends on the anisotropy of the fibers.

When $e = 0$, system (1.2)-(1.3) becomes two decoupled nonlinear Schrödinger equation.

When $e = 1$, system (1.2)-(1.3) is known as Manakov equations. The integrability of this system was proved by Manakov in 1974, which we shall regard it as the *Integrable Manakov System(IMS).*

Equations (1.2)-(1.3) are important for a number of physical applications(see [1-7]) when e is positive and all the remaining constants are set equal to 1. For example, when $e = 2$(Crosignani *et al.* 1982) for two-mode optical fibers; when $e = 2/3$(Menyuk 1987), for propagation of two modes in fibers with strong birefringence, and in the general case $2/3 \le e \le 2$ for elliptical eigen-modes. The special value $e = 1$(IMS) corresponds to at least two possible physical cases, one is the case of a purely electrostrictive nonlinearity, and another is in the elliptical birefringence case, when the angle between the major and minor axes of the birefringence ellipse is approximately 35°. And the experimental observation of Manakov solitons in crystals has been reported by Kang *et al.* (1996). The pulse-pulse collision between wavelength-division-multiplexed channels of optical fiber transmission systems are described by (1.2)-(1.3) with $e = 2$, (Hasewaga and Kodama 1995; Kodama *et al.* 1996; Kodama 1997; Mollenauer *et al.* 1991).

Since the coupled nonlinear Schrödinger (CNLS) equations describe the propagation of light waves in a nonlinear birefringent optical fiber, up to now, they have been studied intensively over 30 years to realize the idea of using optical solitons as information bits in high-speed telecommunication systems(see [8-19]). Moreover, collision of solitary waves is a common phenomena in science and engineering which have diverse applications in many areas of physics, including nonlinear optics, plasma physics, and hydrodynamics.

2 Blow up Condition

Assume that the solution ϕ of (1.1) to satisfy $\phi,\ x\phi,\ \phi_x,\ x\phi_x \in L^2(R^1)$. The domain of integration for each integration appeared in this paper all are global space:$\int = \int_{(0,1)}$.

One easy verifies

$\int |\phi|^2 dx = \int |\phi_0|^2 dx = E_0$.

Let

$E_1 = 6\alpha^2 \int {\phi_x}^2 dx + \gamma(\beta+\gamma) \int |\phi|^6 dx - 6\alpha \int G(|\phi|^2)dx - 3\alpha(\beta$
$+\gamma) Im \int |\phi|^2 \phi_x \bar{\phi} dx$

$$E_2 = \frac{1}{2}\alpha^2 \int {\phi_x}^2 dx + \frac{1}{12}\gamma(\beta+\gamma) \int |\phi|^6 dx - \frac{1}{2}\alpha \int G(|\phi|^2)dx - \frac{1}{4}\alpha(\beta+\gamma) Im \int |\phi|^2 \phi_x \bar{\phi} dx$$

$$E = \frac{\alpha^2}{2} \int |\phi_x|^2 dx + \frac{\mu^2}{6} \int |\phi|^6 dx + \frac{\alpha}{2} \int G(|\phi|^2)dx - \frac{\alpha\mu}{2} Im \int |\phi|^2 \phi_x \bar{\phi} dx.$$

$$y(t) = \int |x|^2 |\phi|^2 dx,$$

$$y'' = (\int |x|^2 2Re\phi_t \bar{\phi} dx)'$$

$$= (2\int x^2[-\alpha Im(\phi_{xx}\bar{\phi}) + \gamma Re(|\phi|^2\phi_x\bar{\phi}) + \beta Re(\phi^2 \bar{\phi_x}\bar{\phi})]dx)'$$

$$= (2\int x^2\{-\alpha Im[(\phi_x\bar{\phi})_x] + \frac{1}{2}\gamma|\phi|^2(|\phi|^2)_x + \frac{\beta}{2}|\phi|^2(|\phi|^2)_x\}dx)'$$

$$= (2\int x^2\{-\alpha Im[(\phi_x\bar{\phi})_x] + \frac{1}{4}(\gamma+\beta)(|\phi|^4)_x\}dx)'$$

$$= 2\int x^2\{-\alpha Im[(\phi_{xt}\bar{\phi} + \phi_x\bar{\phi}_t)_x] + \frac{1}{4}(\gamma+\beta)[2|\phi|^2(|\phi|^2)_t]_x\}dx$$

$$= 2\int x^2\{-\alpha Im[\phi_{xxt}\bar{\phi} + \phi_{xx}\bar{\phi}_t] + \frac{1}{2}(\gamma+\beta)[|\phi|^2(|\phi|^2)_t]_x\}dx$$

$$= -2\alpha Im\int x^2(\phi_{xxt}\bar{\phi} + \phi_{xx}\bar{\phi}_t) + (\gamma+\beta)\int x^2[|\phi|^2(|\phi|^2)_t]_x\}dx$$

$$= 2\alpha Im\int \phi_{xt}(2x\bar{\phi} + x^2\bar{\phi_x}) - 2\alpha Im\int x^2\phi_{xx}\bar{\phi}_t dx - 2(\gamma+\beta)\int x|\phi|^2(|\phi|^2)_t dx$$

$$= -2\alpha Im\int (2\bar{\phi} + 4x\bar{\phi_x} + x^2\bar{\phi_{xx}})\phi_t - 2\alpha Im\int x^2\phi_{xx}\bar{\phi}_t dx - 2(\gamma+\beta)\int x|\phi|^2(|\phi|^2)_t dx$$

$$= -4\alpha Im\int (\bar{\phi}\phi_t + 2x\bar{\phi_x}\phi_t) - 2(\gamma+\beta)\int x|\phi|^2(|\phi|^2)_t dx,$$

$$y^{''} = -4\alpha Im\int (\bar{\phi}\phi_t + 2x\bar{\phi}_x\phi_t) - 2(\beta+\gamma)\int x|\phi|^2(|\phi|^2)_t dx$$

$$y_{tt} = -4\alpha Im\int \bar{\phi}\phi_t dx - 8\alpha Im\int x\bar{\phi}_x\phi_t dx - 4(\beta+\gamma) Re\int x|\phi|^2\bar{\phi}\phi_t dx$$

$$2\alpha Im\int \bar{\phi}\phi_t dx = 2\alpha Im\int \bar{\phi}\{i\alpha x\phi_{xx} + \gamma|\phi|^2\phi_x + \beta\phi^2\bar{\phi}_x + ig(|\phi|^2)\phi\}dx$$

$$= 2\alpha^2 Re\int \bar{\phi}\phi_{xx} dx + 2\alpha\int g(|\phi|^2)|\phi|^2 dx + 2\alpha Im\int (\gamma|\phi|^2\phi_x\bar{\phi} + \beta|\phi|^2\phi\bar{\phi}_x)dx$$

$$= -2\alpha^2\int |\phi_x|^2 dx + 2\alpha\int g(|\phi|^2)|\phi|^2 dx + 2\alpha(\gamma-\beta) Im\int |\phi|^2\phi_x\bar{\phi} dx$$

$= 2\alpha \int g(|\phi|^2)|\phi|^2 dx + 2\alpha(\gamma-\beta) Im \int |\phi|^2 \phi_x \bar{\phi} dx - 2\alpha^2 \int |\phi_x|^2 dx$

$4\alpha Im \int x\bar{\phi}_x \phi_t dx = 4\alpha Im \int x\bar{\phi}_x \{i\alpha\phi_{xx} + \gamma|\phi|^2\phi_x + \beta\phi^2\bar{\phi}_x + ig(|\phi|^2)\phi\} dx$

$= 4\alpha^2 Re \int x\bar{\phi}_x \phi_{xx} dx + 4\alpha Re \int x g(|\phi|^2)\phi\bar{\phi}_x dx + 4\alpha\beta Im \int x\phi^2\bar{\phi}_x^2 dx$

$= -2\alpha^2 \int |\phi_x|^2 dx + 2\alpha \int x g(|\phi|^2)(|\phi|^2)_x + 4\alpha\beta Im \int x\phi^2\bar{\phi}_x^2 dx$

$= -2\alpha^2 \int |\phi_x|^2 dx - 2\alpha \int G(|\phi|^2) dx + 4\alpha\beta Im \int x\phi^2\bar{\phi}_x^2 dx,$

where $G'(s) = g(s)$.
$-2(\beta+\gamma) Re \int x|\phi|^2\bar{\phi}\phi_t dx = -2(\beta+\gamma) Re \int x|\phi|^2\bar{\phi}(i\alpha\phi_{xx} + \gamma|\phi|^2\phi_x + \beta\phi^2\bar{\phi}_x + ig(|\phi|^2)\phi) dx$

$= 2\alpha(\beta+\gamma) Im \int x|\phi|^2\bar{\phi}\phi_{xx} dx - 2(\beta+\gamma)^2 Re \int x|\phi|^4\bar{\phi}\phi_x dx$

$= 2\alpha(\beta+\gamma) Im \int x|\phi|^2(\bar{\phi}\phi_x)_x dx - 2(\beta+\gamma)^2 \int x|\phi|^4 \frac{(|\phi|^2)_x}{2} dx$
$= 2\alpha(\beta+\gamma) Im \int x|\phi|^2\bar{\phi}\phi_{xx} - (\beta+\gamma)^2 \int x|\phi|^4(|\phi|^2)_x dx$

$= 2\alpha(\beta+\gamma) Im \int x|\phi|^2\bar{\phi}\phi_{xx} dx - \frac{(\beta+\gamma)^2}{3} \int x(|\phi|^6)_x dx$

$= \frac{(\beta+\gamma)^2}{3} \int |\phi|^6 dx + 2\alpha(\beta+\gamma) Im \int x|\phi|^2\bar{\phi}\phi_{xx} dx$

$= \frac{(\beta+\gamma)^2}{3} \int |\phi|^6 dx - 2\alpha(\beta+\gamma) Im \int |\phi|^2\phi_x\bar{\phi} dx - 2\alpha(\beta+\gamma) Im \int x(|\phi|^2\bar{\phi})_x\phi_x dx$

$\frac{1}{2}y_{tt} = -2\alpha \int g(|\phi|^2)|\phi|^2 dx - 2\alpha(\gamma-\beta) Im \int |\phi|^2\phi_x\bar{\phi} dx + 4\alpha^2 \int |\phi_x|^2 dx + 2\alpha \int G(|\phi|^2) dx - 4\alpha\beta Im \int x\phi^2\bar{\phi}_x^2 dx + \frac{1}{3}(\beta+\gamma)^2 \int |\phi|^6 dx + 2\alpha(\beta+\gamma) Im \int x|\phi|^2\bar{\phi}\phi_{xx} dx$

$\frac{1}{2}y_{tt} = -2\alpha \int g(|\phi|^2)|\phi|^2 dx + 2\alpha \int G(|\phi|^2) dx - 2\alpha(\gamma-\beta) Im \int |\phi|^2\phi_x\bar{\phi} dx + 4\alpha^2 \int |\phi_x|^2 dx - 4\alpha\beta Im \int x\phi^2\bar{\phi}_x^2 dx + \frac{1}{3}(\beta+\gamma)^2 \int |\phi|^6 dx - 2\alpha(\beta+\gamma) Im \int |\phi|^2\phi_x\bar{\phi} dx - 2\alpha(\beta+\gamma) Im \int x(|\phi|^2\bar{\phi})_x\phi_x dx$

$= -2\alpha \int g(|\phi|^2)|\phi|^2 dx + 2\alpha \int G(|\phi|^2) dx - 4\alpha\gamma Im \int |\phi|^2\phi_x\bar{\phi} dx + 4\alpha^2 \int |\phi_x|^2 dx + \frac{1}{3}(\beta+\gamma)^2 \int |\phi|^6 dx - 4\alpha\beta Im \int x\phi^2\bar{\phi}_x^2 dx - 2\alpha(\beta+\gamma) Im \int x(|\phi|^2)_x\bar{\phi}\phi_x dx$

$E_1 = 6\alpha^2 \int {\phi_x}^2 dx + \beta(\beta+\gamma) \int |\phi|^6 dx - 6\alpha \int G(|\phi|^2) dx - 3\alpha(\beta+\gamma) Im \int |\phi|^2\phi_x\bar{\phi} dx$

$Im \int |\phi|^2\phi_x\bar{\phi} dx = \frac{-E_1}{3\alpha(\beta+\gamma)} + \frac{2\alpha}{\beta+\gamma} \int {\phi_x}^2 dx + \frac{\beta}{3\alpha} \int |\phi|^6 dx - \frac{2}{\beta+\gamma} \int G(|\phi|^2) dx$

$\frac{1}{2}y_{tt} = -2\alpha \int g(|\phi|^2)|\phi|^2 dx + 2\alpha \int G(|\phi|^2) dx + \frac{4\gamma E_1}{3(\beta+\gamma)} - \frac{8\alpha^2\gamma}{\beta+\gamma} \int {\phi_x}^2 dx - \frac{4}{3}\beta\gamma \int |\phi|^6 dx + \frac{8\alpha\gamma}{\beta+\gamma} \int G(|\phi|^2) dx + 4\alpha^2 \int |\phi_x|^2 dx + \frac{1}{3}(\beta+\gamma)^2 \int |\phi|^6 dx - 4\alpha\beta Im \int x\phi^2\bar{\phi}_x^2 dx - 2\alpha(\beta+\gamma) Im \int x(|\phi|^2)_x\phi_x\bar{\phi} dx$

$\frac{1}{2}y_{tt} = -2\alpha \int g(|\phi|^2)|\phi|^2 dx + 2\alpha(1 + \frac{4\gamma}{\beta+\gamma}) \int G(|\phi|^2) dx + 4\alpha^2(1 - \frac{2\gamma}{\gamma+\beta}) \int |\phi_x|^2 dx + (\frac{4}{3}\beta\gamma - \frac{1}{3}(\beta+\gamma)^2) \int |\phi|^6 dx - \frac{4\gamma E_1}{3(\gamma+\beta)} - 4\alpha\beta Im \int x\phi^2\bar{\phi}_x^2 dx + 2\alpha(\beta+\gamma) Im \int x\bar{\phi}^2\phi_x^2 dx$

$y_{tt} = -4\alpha \int g(|\phi|^2)|\phi|^2 dx + 4\alpha(1 + \frac{4\gamma}{\beta+\gamma}) \int G(|\phi|^2) dx + 8\alpha^2(1 - \frac{2\gamma}{\gamma+\beta}) \int |\phi_x|^2 dx + (\frac{8}{3}\beta\gamma - \frac{2}{3}(\beta+\gamma)^2) \int |\phi|^6 dx + \frac{8\gamma E_1}{3(\gamma+\beta)} + 4(\gamma-\beta)\alpha\beta Im \int x\phi^2\bar{\phi}_x^2 dx.$

When $\gamma = \beta$, $y_{tt} = -4\alpha \int g(|\phi|^2)|\phi|^2 dx + 12\alpha \int G(|\phi|^2) dx + \frac{4E_1}{3}$.

When $g = |\phi|^4$,

$y_{tt} = [-4\alpha + \frac{4}{3}\alpha(1 + \frac{4\gamma}{(\beta+\gamma)}) + (\frac{8}{3}\beta\gamma - \frac{2}{3}(\beta+\gamma)^2)] \int |\phi|^6 dx + 8\alpha^2(1 - \frac{\alpha\gamma}{\gamma+\beta}) \int |\phi_x|^2 dx + \frac{8\gamma E_1}{3(\gamma+\beta)} + 4(\gamma - \beta)\alpha\beta Im \int x\phi^2 \bar{\phi}_x^2 dx.$

When $\gamma = \beta = \mu$ and $g = |\phi|^4$, $y_{tt} = \frac{4E_1}{3} = 16E_2 = 16E$.

3 The Conclusion

In this study we consider the case $\gamma = \beta$ and $g(|\phi|^2) = |\phi|^4$.

Theorem 3.1. If $\phi(x,t) \in C(0,\infty; H^1(0,1))$ is

$$\begin{array}{lll} \phi_t & = & i\alpha\phi_{xx} + \beta\phi^2\bar{\phi_x} + \beta|\phi|^2\phi_x + i|\phi|^4\phi, \\ \phi(0,x) & = & \phi_0(x). \end{array} \tag{3.1}$$

a solution with period 1. Then

i) when $E < 0$, ϕ will blow up at finite rime in H^1;

ii) when $E \geq 0$ and $\phi_0 \in L^6$ then ϕ is a global solution in H^1.

Where the H^1 appeared in this section stands for $\sqrt{\int_{(0,1)}(|\phi|^2 + |\phi_x|^2)dx}$

$E = \frac{\alpha^2}{2}\int |\phi_x|^2 dx + \frac{\beta^2}{6}\int |\phi|^6 dx + \frac{\alpha}{2}\int |\phi|^6 dx - \frac{\alpha\beta}{2} Im \int |\phi|^2 \phi_x \bar{\phi} dx.$

Proof. i) $\int |\phi_0|^2 dx = \int |\phi|^2 dx = \int 2x\Re(\bar{\phi_x}\phi)dx$

$\leq 2(\int x^2|\phi|^2 dx)^{\frac{1}{2}} (\int |\phi_x|^2 dx)^{\frac{1}{2}}$

We consider $y(t) = \int |x|^2|\phi|^2 dx > o$, according to Section 2, $y" < 0$ implies that exists is $t_1 > 0$ such that $y(t) \to 0$ as $t \to t_1$. Thus we have

$\|\phi\|_{H^1} > (\int |\phi_x|^2 dx)^{\frac{1}{2}} \to \infty, t \to t_1$.

ii) when $E \geq 0$ and $\phi_0 \in L^6$, $\int |\phi_x|^2 dx$, $\int |\phi|^6 dx$ uniformly bounded for all $t \geq 0$.

Example 3.2. Taking $\phi_0 = \cos(2\pi x + c)) + i\sin(2\pi x + c))$One easy verifies

$\phi = \cos\big(2\pi x + (4\alpha\pi^2 + 1)t + c)\big) + i\sin\big(2\pi x + (4\alpha\pi^2 + 1)t + c)\big)$ is a global solution to (3.1) and $E = 2\pi^2\alpha^2 + \frac{1}{6}\beta^2 + \frac{1}{2}\alpha - \pi\alpha\beta > 0, (\alpha > 0)$.

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Competing Interests

Author has declared that no competing interests exist.

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