Overcoming Effects from Environmental Temperature on the Natural Frequencies of Cable-strut Structures

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Authors contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

Original Research Article

ABSTRACT

The changes in dynamic properties such as natural frequencies and mode shapes are used in vibration health monitoring as tools for assessing the structures health status. They are, however, also affected by environmental conditions like wind, humidity and temperature changes. Of particular importance is the change of the environmental temperature, and it is the most commonly considered environmental variable that influences the vibration health monitoring algorithms. This paper discusses how cable-strut structures can be designed such that their first natural frequency is less sensitive to the temperature changes. The optimization problem is solved by using a genetic algorithm. The level of pre-stress can be regulated to achieve the solution, particularly when a symmetric self-stress vector is chosen.

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1   INTRODUCTION

Cable-strut assemblies are inherently non-linear structure. In literature, they are sometimes called tensegrity structures when some conditions are satisfied i.g, if the structure find its stiffness and self-equilibrium states from the integrity between tension and compression and the stress unilateral property of the components (struts in compression and cables in tension) is satisfied. Fundamentals of tensegrity structures, such as definitions regarding self-stress vectors and its derivation, may be found at [1, 2]. Their stiffness can be improved by regulating the level of pre-stress, using their lowest natural frequencies as indicators for their pre-stress [3]. In vibration health monitoring ('VHM'), the lowest natural frequencies are used as indicators of damage presence [4]. It is well known that, the sensitivity of the natural frequencies to damage is the core of VHM. However, the natural frequencies are also affected by environmental conditions like wind, humidity and temperature changes. Of particular importance is the change of the environmental temperature, and this is the most commonly considered environmental variable influencing the VHM algorithms [5].

To avoid confusion when interpreting the results from VHM algorithms, there are two options: a prior understanding and knowledge of the behaviour of the healthy structure with temperature changes, or optimizing the design of the structure such that its natural frequencies are very little sensitive to the temperature changes. Most of the studies found in literature show that an increase in temperature leads to a decrease

in structural frequencies. [6] reviewed the effect of temperature on vibration properties of civil structures and gave some case studies. They concluded that an increase in temperature leads to a decrease in natural frequencies due to reduced pre-stressed stiffness. Their conclusion is in agreement with other studies regarding the relation between the temperature changes and natural frequencies.

In this paper we investigate how a cable-strut structure can be optimized such that its lowest natural frequencies have a very low sensitivity to the temperature changes.

The effect of temperature changes on the natural frequencies depends on the material used for components, support conditions and the size and shape of the structure. In general, the temperature variations change the natural frequency through the change in size (expansion coefficients) and the change in the elastic modulus [6].

In literature, optimization problems for these structures are normally classified into size, shape and topology optimization [7]. However, the optimum solution here was achieved by using a symmetric self-stress vector, where the level of pre-stress is regulated such that the natural frequencies are not affected by temperature changes. This method has been explained considering a simple 2D string and tube assembly shown in Fig. 1. and a 3D T3 cable-strut (Tensegrity prism), Fig. 2(b) for a certain (pre-decided) topology, shape, and size of the structure.
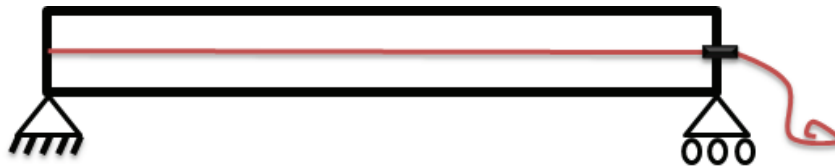

Fig. 1. String and tube assembly

2   FORMULATION STRATEGY

In this study, for complex structures like the one shown in Fig. 2(b), a feasible symmetric unit self-stress vector $\hat{g}$ is chosen by means of one of the Form-Finding methods available in literature [8, 9], to name but a few. For simple structures like the tube and string shown in Fig. 1. this can be done by inspection. The unit self-stress vector $\hat{g}$ can be scaled to regulate the level of pre-stress by means of the scaler ψ. Hence, the designed

self-stress vector g will be seen as:

$$g = \psi \hat{g}, \tag{2.1}$$

where ψ is the level of pre-stress and $\hat{g}$ represents the force density coefficients in each component of the structure. In other words, $\hat{g}$ represents the internal force distribution pattern in the whole structure. In this study, the level of pre-stress represented by ψ is the optimization variable.
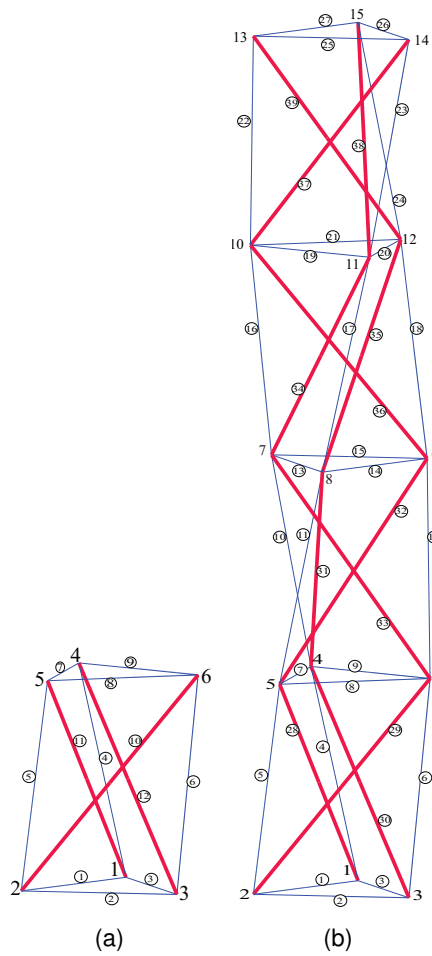

Fig. 2. Topology and the numbering scheme of, (a) The base module of the structure (b) Four-module structure

3 FINITE ELEMENT FORMU-LATION

The finite element model used in this study represents both axial and transversal frequencies of the components, as Euler-Bernoulli beam elements were used. The geometric non-linearity of cable-strut structures is emphasised by the coupling between the axial and bending stiffnesses of the components which has been included in the tangent stiffness matrix $k(T) = k_E(T) + k_G(T)$, with a linear part k_E and a geometric part k_G.

The influence of temperature changes on the dynamic properties of cable-strut structures is a complex topic, as many factors are interacting in this process. These are mainly related to a thermal expansion coefficient α and the variation of the elastic modulus E. In this study, we have made some assumptions concerning the design. The final geometry and topology of the structure are known in advance, the final geometry is connected to a specific pre-stress state, i.e., components are shortened or lengthened from an unstrained length $L_i^o(T)$, introducing axial forces when brought to their design lengths. Two parameters are affected by the temperature changes, with the following assumptions:

(i) The unstrained length $L_i^o(T)$,

$$L_i^o(T) = L_i^o(T_o)[1 + \alpha\Delta T], \qquad (3.1)$$

where $L_i^o(T_o)$ is calculated from the self-stress vector g at a reference temperature T_o, Eq. (2.1). Each element in g is, $g_i = N_i/L_i^f$, [10], where N_i is the axial internal force and L_i^f is the designed length of the component i (pre-defined). But $N_i = EA_i[L_i^f - L_i^o(T_o)]/L_i^o(T_o)$ with $N_i = g_iL_i^f$, from which the unstrained length $L_i^o(T_o)$ can be calculated as $L_i^o(T_o) = EA_iL_i^f/(N_i + EA_i)$. Thus, $L_i^o(T)$ at each temperature change ΔT can be found from Eq. (3.1). (ii) The change of the elastic modulus E with temperature. In literature this topic has been investigated by many researchers [11, 12, 13, 14, 15]. In this study, the experimental results from [14] were adopted. Therefore, it was assumed that the materials were different steel grades. The temperature dependence of the elastic modulus

was thereby assumed as

$$E(T) = E_{20}(-0.000835T + 1.0167), \qquad (3.2)$$

where E_{20} is the elastic modulus at the design temperature, $T_o = 20$ °C, and centigrade are used for temperatures.

Both $L_i^o(T)$, Eq. (3.1) and $E(T)$, Eq.(3.2), were used to evaluate the element tangent stiffness matrix k_T and to iteratively find the corresponding equilibrium state using the Newton-Raphson method [16, 17]. The range for ΔT considered was $\Delta T_{min} = -45 \le \Delta T \le \Delta T_{max} = 25$ °C and $T = T_o + \Delta T$.

The linear beam element mass matrix was calculated at T_o and not modified for ΔT. More details about the finite element model used can be found in [18, 19].

With the tangent stiffness and mass matrices assembled formally as $K(T) = \sum k(T)$ and $M = \sum m$, the natural frequencies of the undamped free vibration of the structure around the evaluated equilibrium state were obtained from the generalized eigenproblem

$$-\omega_k^2 M\phi_k + K_T\phi_k = 0, \qquad (3.3)$$

where ω_k^2 is one of the n eigenvalues and ϕ_k the corresponding eigenvector, with n the number of active degrees of freedom. The eigenvalues were ordered so that $\omega_1 \le \omega_2 \cdots \le \omega_n$. The spectral decomposition thereby gave n natural frequencies ω_k and the related vibration modes ϕ_k of the structure at the considered equilibrium state. As the cable-strut structures normally contain high degrees of symmetry, the resonance solutions will normally contain sets of closely situated frequencies, and possibly eigenspaces of higher dimensions [20].

4 OPTIMIZATION PROBLEM

The optimum value of the level of pre-stress ψ in Eq. (2.1), has been found using built in functions for a genetic algorithm (GA) of Matlab[1]. We minimized the difference between the first natural frequency ω_1 at three successive points of ΔT, chosen as -45, 0 and 25 °C (equivalent to $T = -25, 20$ and 45 °C), Eq. (4.1).

Table 1. Genetic algorithm setup and the basic parameters used in this study

Parameter name	Type and value
Bounds of variables	To be positive
Population (type,size)	Double vector, 50
Selection (function)	Stochastic uniform
Crossover (type,ratio)	Heuristic, default value=1.2
Mutation (function)	Adaptive feasible
Stopping criterion	The number of generation

This is because there is a high probability for the natural frequency to change in between these two points if only ΔT_{min} and ΔT_{max} are chosen because of the non-linearity of this relation. The optimization problem was formulated as:

$$\text{minimize} \sqrt{(\omega_1^{T_1} - \omega_1^{T_2})^2 + (\omega_1^{T_2} - \omega_1^{T_3})^2}$$
$$g = \psi \hat{g} \qquad (4.1)$$
$$\text{subject to} \quad (\psi) > 0,$$

where $\omega_1^{T_x}$ is the first natural frequency at T_x temperature, chosen between ΔT_{min} and ΔT_{max} as mentioned above.

The only constraint applied was that the level of pre-stress had to be positive. However, the level of pre-stress has to be at least enough for the stability of the structure, otherwise the tangent stiffness matrix will be singular. The tangent stiffness matrix singularity was handled by assigning a large fitness value (estimated from multiple runs) to solutions giving singularity.

The performance and efficiency of a genetic algorithm depends on some basic setups and parameters, the used ones are given in Table 1. Different results in different runs were obtained when solving the optimization problem. For this reason we have run the algorithm several times. The solution given in each example is from a typical run, which we believe is converged to a global optimum.

5 NUMERICAL EXAMPLES AND RESULTS

Two examples were considered: a 2D tube and string assembly, Fig. 1., and a 3D cable-strut structure (T3 tensegrity prism), Fig. 2.(b). The method was applied for both structures, where a unit symmetric self-stress vector was used with the level of pre-stress ψ as free parameter for the optimization problem.

The focus here was to maintain the first natural frequency approximately the same as its value f_{20}, the natural frequency at $\Delta T = 0$ ($T = 20$). In the presented Figures, the first four natural frequencies f_T at temperature T were normalized to their values f_{20}.

Cable-strut components can be made from same or different materials. Because of different mechanical properties requirements of tension and compression components, normally different materials for cables and bars are used. Hence, we have investigated the case when different materials are used for bars and cables. Two material combinations were used. The first one was: $\rho = 7585$ kg/m^3, $\alpha_c = 9 \times 10^{-6}/$°C and $E_{20} = 190$ GPa, for cables, and $\rho = 7585$ kg/m^3, $\alpha_b = 11.5 \times 10^{-6}/$°C and $E_{20} = 210$ GPa, for bars. In the second material combination, we switched the material between cables and bars, and investigated the cases; $\alpha_{cable} < \alpha_{bar}$ and $\alpha_{cable} > \alpha_{bar}$.

5.1 Example 1, a Tube and Cable Assembly

The assembly in Fig. 1. was composed of a massive circular cable with a diameter of 0.015 m and a tube (representing the bar action in a cable-strut structure) with thickness of 0.001 m and an outside diameter of 0.05 m with a designed length of 1 m. The level of pre-stress will be regulated through the scaler ψ until the optimum design is reached. The unique unit self-stress vector

for this assembly is $\hat{g} = [0.7071 \quad -0.7071]^T$ with the cable as component 1. With a level of pre-stress of $\psi = 40$ kN/m, the first natural frequency is affected by temperature changes for both material combinations, as shown in Figs. 3.(a), 3.(b). For the first material combination ($\alpha_{cable} < \alpha_{bar}$), the optimum solution was found at $\psi = 37.56$ kN/m. When using the second material combination ($\alpha_{cable} > \alpha_{bar}$) the optimum solution was found to be at $\psi = 59.65$ kN/m, Figs. 4.(a), 4.(b).

5.2 Example 2, a 3-D four-module cable-strut structure

The 3D cable-strut structure (T3 tensegrity prism), Fig. 2.(b), was made up of 12 bars and 27 cables. Its nodal coordinates are listed in Table 2. All bars and cables used in the simulation were massively circular with diameters of 0.065 m and 0.015 m, respectively. For support conditions we assumed node 1 as completely fixed, node 2 as fixed in the Y and Z directions and node 3 as fixed in the Z direction.
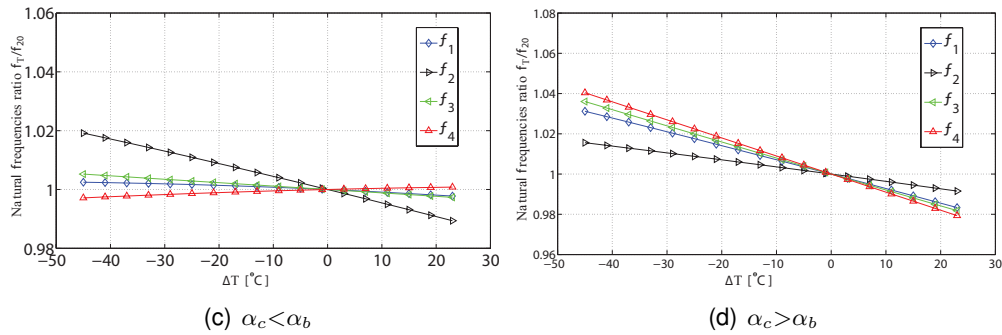

(c) $\alpha_c < \alpha_b$

(d) $\alpha_c > \alpha_b$

Fig. 3. The change of the lowest natural frequencies of the assembly shown in Fig. 1., with different relations for α coefficients and pre-decided level of pre-stress of 40 kN/m

(e) $\alpha_c < \alpha_b$ with level of pre-stress of 37.56 kN/m (f) $\alpha_c > \alpha_b$ with level of pre-stress of 59.65 kN/m
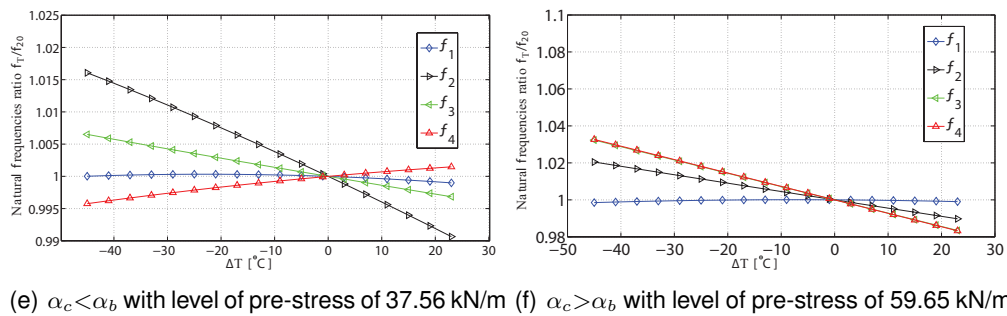
Fig. 4. Optimum design of the assembly shown in Fig. 1., with different relations for α coefficients and optimum level of pre-stress, the first natural frequency is approximately constant with temperature change

In the Form-Finding step of this structure, the equilibrium matrix A is evaluated from a given topology and nodal coordinates for single module, "the base module". A singular value decomposition (SVD) of A gives the independent self-stress states s_b of the base module, from which the self-stress vector g_b in Table 3 of the base module can be evaluated . For the whole structure, a feasible symmetric self-stress vector g was found with values corresponding to the shared components between modules were added. Then, the unit self stress vector $\hat{g}$ was calculated by taking the norm of g. As mentioned above, the unit self-stress vector $\hat{g}$ can be scaled to regulate the level of pre-stress by means of the scaler ψ. Hence, the self stress vector will be seen as in Eq. (2.1). It is worth nothing that evaluated self-stress vector g of the whole structure is symmetric.

An example for non-optimal solution is when $\psi = 0.55$ MN/m for both cases $\alpha_{cable} < \alpha_{bar}$ and $\alpha_{cable} > \alpha_{bar}$, Figs. 5(a) and 5(b), respectively. In both cases the first natural frequency changed with ΔT. It is interesting to observe that for both material combinations the optimum design was found at $\psi = 0.881$ MN/m, Figs. 6(a) and 6(b) using the same self-stress vector g.

Table 2. Nodal coordinates of the cable-strut structure shown in Fig. 2(b)

Node No.	Coordinates [m]		
	X	Y	Z
1	0.50	0.00	0.00
2	-0.25	0.433	0.00
3	-0.25	-0.433	0.00
4	0.433	0.25	2.00
5	-0.433	0.25	2.00
6	0.00	-0.50	2.00
7	0.25	0.433	4.00
8	-0.50	0.00	4.00
9	0.25	-0.43	4.00
10	0.00	0.50	6.00
11	-0.433	-0.25	6.00
12	0.433	-0.25	6.00
13	-0.25	0.433	8.00
14	-0.25	-0.433	8.00
15	0.50	0.00	8.00

Table 3. Force density coefficients of the base module and the whole structure in Fig. 2. (symmetric self-stress)

Component	Base module ($\hat{g}_b$)	Whole structure ($\hat{g}$), optimum at relevant ψ, depends on the material combination used
All side cables	0.3778	0.1853
All bars	-0.4160	-0.2040
All cables forming triangles between modules	—	0.0919
All cables forming top and bottom triangles	0.0937	0.0459

(g) $\alpha_c < \alpha_b$ with level of pre-stress of 0.55 MN/m

(h) $\alpha_c > \alpha_b$ with level of pre-stress of 0.55 MN/m

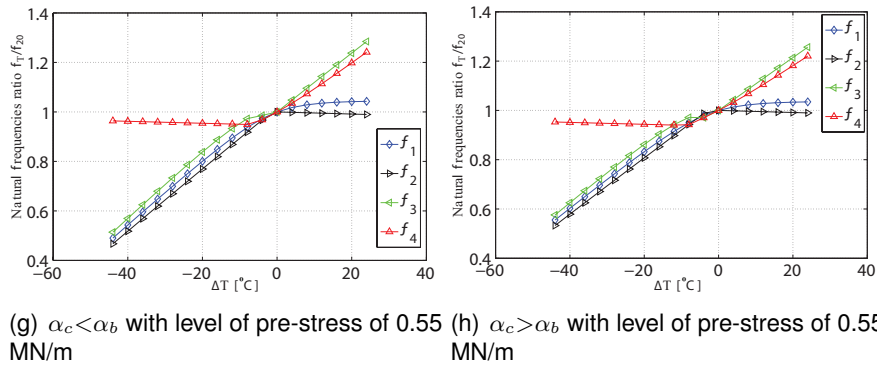**Fig. 5. Non-optimum design of the cable-strut structure shown in Fig. 2(b), the first natural frequency changes with temperature change**

(i) $\alpha_c < \alpha_b$ with level of pre-stress of 0.881 MN/m

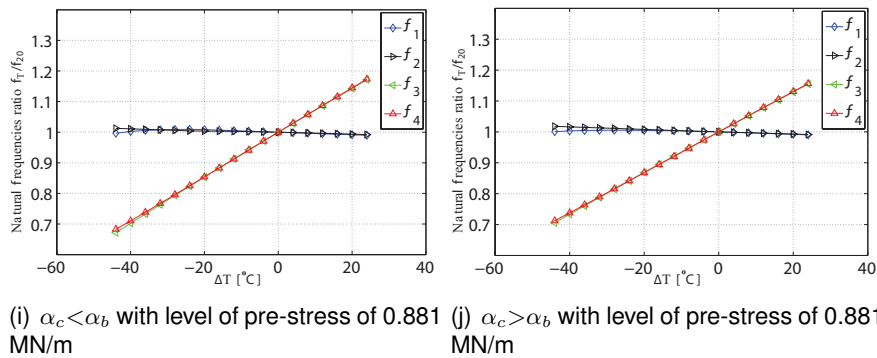(j) $\alpha_c > \alpha_b$ with level of pre-stress of 0.881 MN/m

Fig. 6. Optimum design of the cable-strut structure shown in Fig. 2(b), using the unit self-stress vector $\hat{g}$ from Table 3, and regulating the pre-stress level

6 CONCLUSIONS

In the vibration health monitoring (VHM) methods, a confusing between weather the change in the low natural frequencies was caused by damage or changes in environment temperature is a major issue. In this study, it is shown that cable-strut structures can be designed such that some of their natural frequencies are little sensitive to the temperature changes. The level of pre-stress combined with a symmetric unit self-stress vector can be chosen such that some of its low natural frequencies are not sensitive to the temperature changes. The finding in this study can be very useful when thinking about vibration health monitoring methods of the cable-strut structures. For a future study it might be interested to study if asymmetric self-stress vector can be chosen (if the structure has multiple self stress states) to achieve the same finding.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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